Moderating Noise-Driven Macroeconomic Fluctuations Under Dispersed Information *

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Abstract

Can aggregate noise shocks produce large macroeconomic fluctuations, and if so, is there anything that policymakers can do about them? Yes and yes, if news about idiosyncratic fundamentals is contaminated by aggregate noise. I study a business cycle model where agents with rational expectations receive noisy signals about future productivity. The model features dispersed information, which allows aggregate noise shocks to produce frequent large fluctuations in the capital stock. Because of the information friction, a policymaker with an informational advantage can improve outcomes. I consider policies that affect investment incentives by distorting the intertemporal wedge. I calculate the optimal policy rule, and find that policymakers should discourage investment booms after aggregate news shocks.

JEL-Codes: D84, E21, E32
Keywords: Noise Shocks, Bubbles, Sentiments, Incomplete Information, Heterogeneous Beliefs, Business Cycles, Optimal Policy

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1 Introduction

Business cycles may be driven by non-fundamental noise shocks, featuring large capital expansions and contractions. Do policymakers have any recourse? Not if agents in the model have full information and rational expectations (FIRE). But if the economy features incomplete information, policymakers with an informational advantage can improve outcomes. This is fortunate because incomplete information can also amplify the effects of noise shocks on the business cycle. In this paper I characterize this amplification and the optimal policy response.

I study noise-driven macroeconomic fluctuations in a standard business cycle model augmented with news shocks and dispersed information. To produce such fluctuations, shocks to macroeconomic sentiments are introduced in a standard way: agents receive news about future productivity, but that news is not a perfect forecast, and the error is a "noise shock."[1] When agents receive a noise shock, they behave as if future productivity will improve, but that improvement is never realized. As a result, noise shocks produce movements that can resemble a business cycle, without any changes to measured fundamentals. However in typical models with noise shocks, common information[2] limits the effects of these noise shocks: noise-driven macroeconomic fluctuations cannot be both large and frequent. This is because if the noise shock variance is large, agents realize that their news process is inaccurate, so they do not make large changes to their behavior when they receive news, and noise-driven fluctuations will be small. Alternatively if the noise shock variance is small, then large noise-driven fluctuations are rare.

The information friction dismantles these limits, so that noise shocks can be a large contributor to business cycle volatility if they are sufficiently correlated across individuals. With dispersed information, agents cannot observe the news received by everyone in the economy; they only receive news about their own productivity. Idiosyncratic productivity is much more volatile than aggregate productivity. So for the same noise shock process, agents perceive their news to be more accurate than they would under common information. As a result, they are more elastic to news and to

[1] This structure for noise shocks is typical in the DSGE and VAR literatures, where noise affects agents' forecasts of future productivity, which corrupts their choices of investment and other dynamic variables. Chahrour and Jurado (2018) formally define this type of noise, and show how it is isomorphic to a variety of "news" processes. This contrasts with the typical use of noise in the incomplete information literature, where noise affects agents' now-casts of contemporaneous fundamentals. Lorenzoni (2009) is a classic example, which also features dispersed information and noise shocks; price rigidities rather than capital provide a propagation mechanism that allows noise shocks to have large persistent effects. One of the reasons noise appears in this way across the incomplete information literature is the difficulty of including capital in such models, especially when they feature endogenous signals. I apply the methodology from Adams (2021) to overcome this difficulty.

[2] "Common information" refers to the case where all agents in the economy have the same information set, the terminology used by Lorenzoni (2011) and others. Typically this is equivalent to full information, but in the literature on news and noise they are dissimilar: "full information" is stronger, implying that all agents observe all fundamental shocks.

noise shocks than they would be without the friction. This effect yields a possibility result: if the noise has a large aggregate component, then the macroeconomy can feature large noise-driven fluctuations.

The information friction also introduces a role for policy. Policymakers can improve outcomes if they have more information than individuals. I study policy rules that operate through the investment wedge, such as an investment tax. I calculate optimal rules for policymakers with different information sets and in different models. Across these settings, the optimal policy moderates noise-driven fluctuations by discouraging investment during booms and encouraging it during busts. This is because agents over-respond to the aggregate component of their news process, from the perspective of the policymaker who is able to observe the rest of the macroeconomy. When the policymaker sees aggregate news, they intervene in agents' consumption/savings decision to make the entire macroeconomic response less elastic.

How important are noise-driven fluctuations? A growing empirical literature finds a large role for noise or other non-fundamental shocks in contributing to business cycles. Chahrour and Jurado (2022) use a non-causal VAR to identify noise shocks in US data and find that noise explains 60% of GDP volatility and almost 40% of consumption and stock market volatility at business cycle frequencies. Forni, Gambetti, Lippi, and Sala (2017) employ a more restrictive identification strategy, using data on stock prices and consumer sentiments in a VAR to identify noise shocks, which they estimate as driving 14-24% of GDP volatility and 9-22% of consumption volatility, depending on the specification. Gazzani (2020) uses the same strategy to estimate the effects of noise in the housing market and finds that noise explains 40-50% of short-run housing price volatility, and most of the 2001-2009 American boom and bust. Abstracting from noise specifically, Angeletos, Collard, and Dellas (2020) calculate the reduced form shocks that explain most of business cycle fluctuations; what they identify as the "main business cycle" shock is almost entirely non-fundamental, disconnected from present or future changes to productivity. The dispersed information model that I study allows for noise to be as important as the empirical evidence suggests.

In contrast, existing structural models typically imply that noise shocks are small contributors to business cycle volatility. Chahrour and Jurado (2018) estimate several such models using common data and noise definitions. In the RBC model of Schmitt-Grohé and Uribe (2012), noise contributes only 5% of consumption volatility, while in the New Keynesian model of Barsky and Sims (2012), it is 9%. However there are exceptions: Blanchard, L'Huillier, and Lorenzoni (2013) include a large permanent component to productivity which makes forecasts particularly sensitive to the noise shock, and include additional rigidities in the standard New Keynesian model which makes consumption especially forward looking; together, these ingredients allow noise to contribute to 57% of consumption volatility, although much smaller amounts for output and investment. Typical in these papers is the assumption of common information: agents make noisy forecasts, but they all make the same noisy forecasts.

When I drop this assumption in the following sections, the importance of noise increases by orders of magnitude. This is the first general equilibrium business cycle model with rational expectations and capital where noise shocks drive the majority of output volatility.

As far as I know, this is the first study of the optimal policy response to noise-driven fluctuations in the capital stock caused by noise shocks. Models with noise where agents and policymakers share common information do not have a role for policy (except to resolve other frictions) because agents make the best choices given the available information. In the incomplete information literature, there exist studies of optimal policy to resolve different information frictions, but none consider how to respond to capital fluctuations driven by noise shocks to forecasts of future productivity. Still, some papers come to related conclusions in dissimilar settings. A common pattern in the literature is that agents are too elastic to noisy signals (the classic problem of Morris and Shin (2002)) so the optimal policy is to somehow make agents less elastic to their noisy signal. Angeletos and Pavan (2009) and Lorenzoni (2010) both come to this conclusion considering tax and interest rate policy respectively, in models where agents receive noisy public signals of contemporaneous aggregate productivity that is later revealed to the policymaker. Dupor (2005) studies a business cycle model with capital subject to reduced-form expectations shocks, and also finds that policymakers should act to moderate the resulting noise-driven fluctuations. With the type of structure that I study, aggregate noise shocks can microfound this type of expectations shock, because the noise shocks cause average forecast errors.

This paper fits into several broader literatures. First, it joins the literature studying optimal policy in the context of incomplete information; in addition to those cited already, this includes Adam (2007), Nimark (2008), Baeriswyl and Cornand (2010), Paciello and Wiederholt (2014), Angeletos and La'O (2019), Benhima and Blengini (2020), and Angeletos, Iovino, and La'O (2020) among others. Next, it contributes to a set of papers studying how dispersed information in business cycle models can amplify real shocks, which includes Venkateswaran (2014), Chahrour and Gaballo (2020), and Angeletos and Lian (2020).[3] Finally, this paper joins the long literature studying the effects of non-fundamental shocks in models with incomplete information following Lucas (1972). Angeletos and Lian (2016) provide a recent survey.

The remainder of the paper is organized as follows. Section 2 describes the assumptions of the baseline model. Section 3 describes the noise-driven fluctuations

[3] These papers achieve amplification through other channels than correlated noise shocks. Worker search and matching is the amplification channel in Venkateswaran (2014), where firms are ordinarily more responsive to the idiosyncratic component of productivity than the aggregate component, because their output is sufficiently substitutable; under dispersed information, firms' search and hiring is too responsive to the aggregate component. Chahrour and Gaballo (2020) features amplification due to strategic complementarity in housing investment that affects endogenous signal precision. In Angeletos and Lian (2020) dynamic savings decisions are strategic substitutes, so their model features amplification because savings is more elastic to idiosyncratic discount than aggregate discount factor shocks, and so an information friction makes households too elastic to the aggregate shock.

and other dynamics. Section 4 then considers the optimal policy response. Section 5.2 studies how the importance of noise for the business cycle depends on assumptions about the shock processes and other parameters. Section 6 concludes.

2 Baseline Model

This sections lays out a simple macroeconomic model with dispersed information that contains enough features to illustrate the main conclusions for optimal policy.

The model is a Lucas-style dispersed information economy with standard real business cycle ingredients, similar to Graham and Wright (2010) albeit with a richer shock and information structure in order to generate noise-driven fluctuations.[4] Islands receive news shocks about future productivity, but news contains errors: the noise shock. The information friction is that firms and households on each island can observe prices and quantities on their own island, but not the aggregate economy. Unable to disentangle aggregate noise from idiosyncratic components of their news signals, islands respond more to noise shocks than they would if they had common information.

The model also differs from Graham and Wright (2010) by including additional structure in order to generate standard business cycle correlations. Empirical evidence following Beaudry and Portier (2006) suggests that aggregate news shocks produce comovement: output, consumption, investment, and hours all increase. And VAR evidence studying the noise component specifically also implies comovement after a noise shock (Forni, Gambetti, Lippi, and Sala (2017), Chahrour and Jurado (2022)). I follow Jaimovich and Rebelo (2009) who use investment adjustment costs and capital utilization to generate TFP and output increases after a positive news shock, and preferences with a small wealth effect on labor supply in order to prevent substitution towards leisure.[5]

2.1 Households

There is a continuum of islands $\mathcal{I}$ indexed by i. On each island, there is a unit measure $\lambda(i) = 1$ of identical and infinitely lived households.

[4] In Graham and Wright (2010), the single aggregate shock is to productivity, and the information friction attenuates the economy's response to the shock. In such a model, any business cycles are associated with changes to aggregate productivity, so additional structure is needed to produce fluctuations where aggregate investment moves without any corresponding change in measured fundamentals.

[5] I adopt the approach by Jaimovich and Rebelo (2009) which is suited for a real economy in general equilibrium. But several business cycle theories in other settings incorporate alternative features that achieve such comovement after a news signal, including Jaimovich and Rebelo (2008) in an open economy setting with capital and labor adjustment costs, and Blanchard, L'Huillier, and Lorenzoni (2013) in a New Keynesian model.

The island i representative household's preferences over current and future consumption are represented by the utility function

$$E_{i,t}\left[\sum_{s=0}^{\infty}\beta^s \ln\left(C_{i,t+s}-\chi N_{i,t+s}^{\theta}\right)\right] \tag{1}$$

where $C_{i,t}$ is the household's consumption in period t, β is their discount factor, and θ is their Frisch labor supply elasticity. This utility function corresponds to GHH preferences (Greenwood, Hercowitz, and Huffman, 1988) which eliminates the wealth effect on labor supply, so that hours may increase in response to positive news about future productivity. The expectation operator $E_{i,t}$ is conditional on the representative household i's information set $\Omega_{i,t}$.

Households earn two types of income. They supply $N_{i,t}$ labor on their island, for which they are paid real wage $W_{i,t}$. They also own the capital on their island, $K_{i,t}$, and choose the capital utilization rate $U_{i,t}$. They rent capital services $U_{i,t}K_{i,t}$ to firms at rental rate $R_{K,i,t}$.

The representative household purchases consumption $C_{i,t}$ (the numeraire) and investment $I_{i,t}$ at unit price in an economy-wide market. Therefore their budget constraint is

$$W_{i,t}N_{i,t}+R_{K,i,t}U_{i,t}K_{i,t}=C_{i,t}+I_{i,t} \tag{2}$$

Investment is used to construct new capital. A household owning $K_{i,t}$ capital and investing $I_{i,t}$ faces the law of motion:

$$K_{i,t+1}=I_{i,t}(1-\varphi(\frac{I_{i,t}}{I_{i,t-1}}))+(1-\delta(U_{i,t}))K_{i,t} \tag{3}$$

where φ is a convex functions satisfying $\varphi(1)=0$, $\varphi'(1)=0$, and $\varphi''(1)>0$, the CEE adjustment cost assumptions of Christiano, Eichenbaum, and Evans (2005). $\delta(U_{i,t})$ is the increasing and convex function determining depreciation from the utilization rate.

The household's problem is to choose sequences of $C_{i,t}$, $I_{i,t}$, $U_{i,t}$, and $K_{i,t+1}$ to maximize (1) subject to the budget constraint (2) and law of motion (3). The solution to this problem is characterized by a labor supply equation

$$W_{i,t}=\frac{\chi\theta N_{i,t}^{\theta-1}}{C_{i,t}-\chi N_{i,t}^{\theta}} \tag{4}$$

a first order condition for utilization

$$R_{K,i,t}K_{i,t}=Q_{i,t}\delta'(U_{i,t})K_{i,t} \tag{5}$$

and an Euler equation for capital

$$Q_{i,t}=E_{i,t}\left[\Lambda_{i,t+1}\left(R_{K,i,t+1}U_{i,t+1}+Q_{i,t+1}\left(1-\delta(U_{i,t+1})\right)\right)\right] \tag{6}$$

where expectations $E_{i,t}$ are conditional on representative household i's information set $\Omega_{i,t}$. The household's stochastic discount factor is $\Lambda_{i,t+1} = \beta \frac{C_{i,t} - \chi N_{i,t}^{\theta}}{C_{i,t+1} - \chi N_{i,t+1}^{\theta}}$. On the right-hand side of (6), households discount the real return on their capital, plus the marginal value of units of capital they carry over. Lastly, $Q_{i,t}$ denotes Tobin's marginal Q (the value of a marginal unit of capital) which is determined dynamically by the household's final equilibrium condition:

$$1 = Q_{i,t} \left(1 - \varphi(\frac{I_{i,t}}{I_{i,t-1}}) - \varphi'(\frac{I_{i,t}}{I_{i,t-1}}) \frac{I_{i,t}}{I_{i,t-1}} \right) + E_{i,t} \left[\Lambda_{i,t+1} Q_{i,t+1} \varphi'(\frac{I_{i,t+1}}{I_{i,t}})(\frac{I_{i,t+1}}{I_{i,t}})^2 \right] \quad (7)$$

which is standard for CEE investment. When households expect faster investment growth in the future, they are incentivized to accelerate investment in the present in order to relieve the adjustment cost. This is the mechanism that allows investment to rise upon news of future productivity improvements.

2.2 Firms

There are two types of firms in the economy. There are intermediate goods firms that each operate on an island indexed by i, and there are final goods firms that aggregate the intermediate goods into final goods in an economy-wide market.

Final goods firms aggregate specialized goods Y^i of type $i \in \mathcal{I}$ with a CES production function:

$$Y_t = \left(\int_{i \in \mathcal{I}} e^{\xi_{i,t}} \left(Y_t^i \right)^{\frac{\eta-1}{\eta}} d\lambda(i) \right)^{\frac{\eta}{\eta-1}} \quad (8)$$

with $\eta \neq 1$. $\xi_{i,t}$ is a stochastic island-specific demand shifter.

Final goods can be used for either consumption or investment across islands. Therefore the market clearing condition for the final goods is

$$Y_t = \int_{i \in \mathcal{I}} (C_{i,t} + I_{i,t}) \, d\lambda(i) \quad (9)$$

Consumption is the numeraire, so final goods also have unit price. Final goods firms purchase intermediates at price $P_{i,t}$, so their demand for intermediates is given by the CES demand function

$$P_{i,t} = e^{\xi_{i,t}} (\frac{Y_t}{Y_t^i})^{\frac{1}{\eta}} \quad (10)$$

Intermediate goods firms are perfectly competitive and have constant returns. The representative firm on island i in period t uses specialized capital services $U_{i,t} K_{i,t}$ and labor $L_{i,t}$ with stochastic productivity $A_{i,t}$ to produce output Y_t^i by

$$Y_t^i = A_{i,t} (U_{i,t} K_{i,t})^{\alpha} L_{i,t}^{1-\alpha} \quad (11)$$

Firms rent capital services at rental rate $R_{K,i,t}$ and hire labor at wage $W_{i,t}$ from the households on island i in period t. They sell their output at price $P_{i,t}$. The representative firm chooses inputs to maximize their profits, which implies that labor and capital demands for island i are given by

$$P_{i,t}\alpha\frac{Y_t^i}{U_{i,t}K_{i,t}} = R_{K,i,t} \qquad P_{i,t}(1-\alpha)\frac{Y_t^i}{L_{i,t}} = W_{i,t} \tag{12}$$

2.3 Information

Agents on island i have perfect information about their own island, but not the macroeconomy. They receive three noisy signals that inform them about economic aggregates, but they are buffeted by four fundamental shocks, so they cannot perfectly infer the state of the economy.

The first signal is productivity. In logs, productivity $\ln A_{i,t}$ is the sum of an aggregate component $\ln A_t$ and a mean zero idiosyncratic component $\ln \hat{A}_{i,t}$ satisfying

$$\ln A_{i,t} = \ln A_t + \ln \hat{A}_{i,t} \tag{13}$$

Agents cannot observe aggregate productivity directly, and can only estimate its value based on their island-specific productivity and other signals. The productivity components are independent AR(1) processes:[6]

$$\ln \hat{A}_{i,t} = \rho_a \ln \hat{A}_{i,t-1} + \epsilon_{\hat{a},i,t} \qquad \epsilon_{\hat{a},i,t} \sim N(0,\sigma_{\hat{a}}) \tag{14}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \epsilon_{a,t} \qquad \epsilon_{a,t} \sim N(0,\sigma_a) \tag{15}$$

The news signal is news about future productivity. In period t, agents on island i receive a noisy signal about their productivity shocks κ periods into the future. The news $\nu_{i,t}$ is given by

$$\nu_{i,t} = \epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa} + \zeta_t \tag{16}$$

where ζ_t is the "noise" shock. Noise is the error in the news signal, which prevents islands from perfectly predicting their future productivity. The noise shock ζ_t is an aggregate shock, common to all islands. This is the crucial assumption for the possibility result: noise must be correlated across islands in order to amplify business cycles. Section 5.1 explores how relaxing this assumption affects the amplification result.

The third signal is demand, which provides a noisy signal of aggregate output. Agents observe the demand function for their island's goods, from which they infer the quantity $e^{\xi_{i,t}}Y_t^{\frac{1}{\eta}}$ by equation (10). In logs, the demand signal $D_{i,t}$ is given by

$$\ln D_{i,t} = \xi_{i,t} + \frac{1}{\eta}\ln Y_t \tag{17}$$

[6] I let the autoregressive coefficient ρ_a be the same for both productivity processes. This is not necessary for any of the main conclusions from the model; rather, it eliminates an additional effect of the information friction. When idiosyncratic and aggregate productivity have different autocorrelations, agents in a dispersed information model become uncertain about the persistence of the productivity changes they observe, and will over or under-respond to aggregate shocks.

Islands cannot distinguish between the effects of aggregate output Y_t and the idiosyncratic demand-shifter $\xi_{i,t}$, so $\ln D_{i,t}$ is a noisy signal of the aggregate state. Additionally, demand is an endogenous signal: $\xi_{i,t}$ has an exogenous process, but the process for Y_t is determined in equilibrium by the choices made by households and firms.

An island's information set $\Omega_{i,t}$ includes all of the local endogenous variables on island i plus the three signals, and the information set evolves by

$$\Omega_{i,t} = \{\Omega_{i,t-1}, \nu_{i,t}, \ln A_{i,t}, \ln D_{i,t}, \mathbf{v}_{i,t}\} \tag{18}$$

where $\mathbf{v}_{i,t}$ is the vector of endogenous prices and quantities on island i.

Each island is affected by four exogenous independent stochastic processes: two aggregate processes A_t and ζ_t; and two idiosyncratic processes $\hat{A}_{i,t}$ and $\xi_{i,t}$. However, islands receive only three signals that are informative about these processes: $\nu_{i,t}$, $A_{i,t}$, and $D_{i,t}$. With more shocks than signals, the aggregate state of the economy will not be revealed.

2.4 Equilibrium Definition

Given infinite sequences of exogenous variables $\{A_t, \hat{A}_{i,t}, \zeta_t, \xi_{i,t}\}$ for all $i \in \mathcal{I}$, a competitive equilibrium in this economy consists of infinite sequences of prices, $\{P_{i,t}, W_{i,t}, R_{K,i,t}\}$ for all $i \in \mathcal{I}$; allocations $\{C_{i,t}, I_{i,t}, U_{i,t}, K_{i,t}, L_{i,t}, Y_t^i, Y_t\}$ for all $i \in \mathcal{I}$; and information sets $\Omega_{i,t}$ for all $i \in \mathcal{I}$ such that:

1. Households maximize utility (1), subject to the constraints (2) and (3)
2. Intermediate firms choose allocations to maximize profits, satisfying the production function (11) and factor demands (12).
3. Final goods firms choose allocations to maximize profits, satisfying the production function (8) and input demands (10).
4. The goods markets must clear, satisfying equation (9).
5. Firm productivities are given by (13).
6. News signals are given by (16).
7. Information sets evolve by (18).

2.5 Calibration

I calibrate the model to resemble the US economy (Table 1). One time period represents a year, so I select the discount factor $\beta = 0.95$ to target a 5% steady state annual return. I parameterize depreciation as a constant elasticity function $\upsilon U_{i,t}^{\omega}$.[7] The depreciation function $\delta(U_{i,t})$ is parameterized so that the steady-state depreciation rate is $\delta(\bar{U}) = 0.05$, and the capital share is set to $\alpha = 0.38$, both to match post-war averages from the US NIPA. I calibrate the adjustment cost and labor supply elasticity parameters as in Jaimovich and Rebelo (2009): $\varphi''(1) = 1.3$ and $\theta = 1.4$. It is important that $\eta > 1$ so that islands' outputs are substitutes; in the Cobb-Douglas case of $\eta = 1$, island revenue is unaffected by productivity because any change in output is exactly offset by a change in price. Moreover, when islands' output is not sufficiently substitutable, the Blanchard-Kahn condition is not satisfied, so I select a large value, setting $\eta = 10$. Therefore islands are interpreted as relatively disaggregated industries producing close substitutes, rather than large complementary sectors.

The parameters for the idiosyncratic productivity process (14) are the GMM estimates by Lee and Mukoyama (2015) for US manufacturing plants using plant-level data from the Annual Survey of Manufactures. I use US KLEMS data to estimate the process (15) for aggregate productivity, which crucially has a smaller standard deviation than idiosyncratic productivity. The standard deviation of the idiosyncratic demand shock σ_ξ is not well disciplined by the literature; I choose a moderate value, because a small variance would closely resemble common information while a large value would eliminate the information signal provided by aggregate demand.

The main parameter governing the macroeconomic effects of noise is the standard deviation of noise shocks, σ_ζ. After calibrating the other parameters, I choose $\sigma_\zeta = 0.115$ in order to target the contribution of TFP noise shocks to the business cycle. Specifically, I match the 60% share of aggregate output variance that Chahrour and Jurado (2022) estimate is due to TFP noise shocks. Thus σ_ζ is explicitly chosen to resolve the following puzzle: reduced form studies find a large role for noise shocks in output volatility that is not explained by existing FIRE business cycle theories. One way to interpret this parameter choice is that the calibration of σ_ζ and σ_a imply that only 1% of the variance of aggregate productivity shocks can be anticipated using aggregate data. Lastly, I choose $\kappa = 4$ to be the number of years between a news signal and realized productivity.

This calibration is chosen as conventionally as possible. Still, several parameters are not well informed by prior research, in particular σ_ζ, σ_ξ, η, and κ. Therefore in Section 5.2 I explore the model results depend on these parameter choices. In general, adjusting these parameter values do not change the main qualitative conclusions.

I linearize the equilibrium conditions and solve the model using Signal Operator Iteration (Adams, 2021). The linearized model is reported in Appendix A.

[7] The constant elasticity depreciation function has only one parameter to be chosen: υ pins down the steady-state depreciation rate. The remaining steady state equations determine the depreciation elasticity, $\omega = 2.05$.

Parameter	Interpretation	Value
β	Discount factor	0.95
α	Capital share	0.38
η	Elasticity of substitution	10
θ	Labor supply elasticity	1.4
$\delta(\bar{U})$	Steady-State Depreciation	0.05
$\varphi''(1)$	Investment Adjustment Cost	1.3
ρ_a	Persistence of technology shock	0.84
$\sigma_{\hat{a}}$	Standard deviation of idiosyncratic technology shock	0.30
σ_a	Standard deviation of aggregate technology shock	0.013
σ_ζ	Standard deviation of aggregate noise shock	0.115
σ_ξ	Standard deviation of idiosyncratic demand shock	0.10
κ	Periods between news and realization	4

Table 1: Baseline calibration

3 Equilibrium Analysis

In this section I examine how the macroeconomy experiences noise-driven fluctuations in equilibrium. First I describe how agents respond to information. Second, I examine how aggregate shocks affect the macroeconomy. Third, I study how the information friction exacerbates the fluctuations. Finally, I describe how the information friction amplifies business cycle volatility in general.

3.1 Decision-making

Agents observe three noisy signals: news about future productivity, their current realized productivity, and demand for their goods. But agents are affected by four independent fundamental shocks, so these signals cannot be perfectly revealing. Figure 1 plots islands' impulse responses to a unit innovation of the news signal. These are not the response to any single shock, which agents cannot observe. Rather, they are the responses to learning new information, i.e. a forecast error that is a linear combination of many fundamental shocks.

When agents receive news that their productivity is likely to improve, they want to immediately increase consumption because they expect future income to rise. They also want to increase investment, in order to take advantage of higher future returns on capital. To meet both of these desires, output must rise. This is possible because the GHH preferences eliminate any wealth effect on labor supply, which is determined entirely by the wage. Productivity and capital will only increase with a delay, so the behavior of capital utilization determines how the marginal product of labor and wage respond on impact.

Why does utilization increase after a news innovation? The marginal cost of cap-

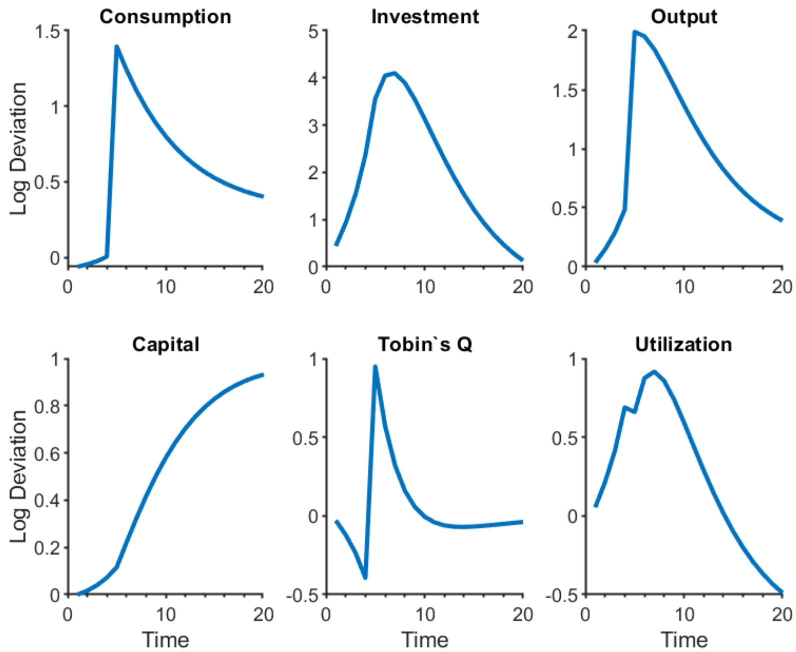


Figure 1: News Innovation Impulse Responses

ital utilization is the value of the marginal depreciation of the capital stock. The marginal value of capital falls on impact, so the cost of utilization also falls. This decline in the marginal value of capital is due to the CEE form of the investment adjustment costs, which implies that the marginal value of capital falls when investment accelerates. Because utilization rises, output rises, the marginal product of labor rises, and so does labor itself. Thus the islands experience comovement after a positive news innovation: consumption, investment, output, labor, and utilization all increase.

On average, the news is followed by a future increase in productivity, so after 4 periods consumption, investment, hours, and output all rise further. Capital accumulates over time, propagating the boom.

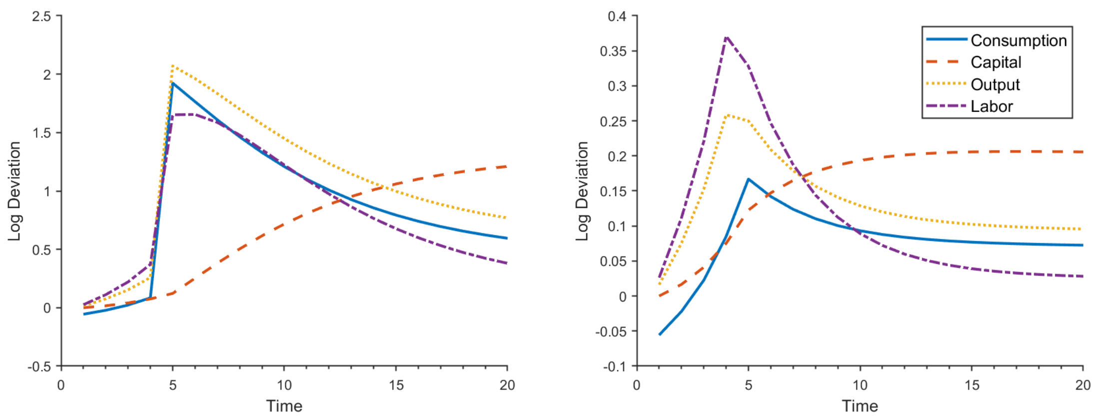
However, this is only the average response. Sometimes the news signal is driven by a future productivity shock, but sometimes it is driven by a noise shock. After 4 periods, the islands find out if their productivity behaves as predicted. If not, they over- or under-invested. For an individual this risk is rare: the news signal is a relatively accurate predictor of island-specific productivity. Therefore agents trust their signals and an island's response to news resembles that in Jaimovich and Rebelo (2009).

But the aggregate economy behaves very differently than an individual island, because while the forecast error on productivity is small for any given island, the errors are correlated across islands and relatively large in the aggregate.

3.2 Aggregate Dynamics

There are two aggregate shocks that can move the macroeconomy: the shock to aggregate productivity and the noise shock. The remaining shocks - demand innovations and island-specific productivity - are purely idiosyncratic and cannot move the macroeconomy.

A positive aggregate productivity shock first generates a news innovation on all islands. Aggregate consumption, investment, and output increase (Figure 2, panel (a)). After four periods, productivity increases, raising aggregate output and other series again. This economic boom is even larger than the average response to a news signal in Figure 1 for two reasons. First, the productivity improvement is more than islands expect, because news is not a perfect forecast of productivity. Second, the productivity improvement is economy-wide, increasing demand from other islands relative to a purely idiosyncratic shock. The resulting aggregate patterns resemble a productivity improvement in a typical business cycle model with news.



(a) Aggregate Productivity Shock (b) Aggregate Noise Shock

Figure 2: Fundamental Shock Impulse Responses

An aggregate noise shock is a common error in all islands' news signals. On impact, it is exactly the same as an aggregate productivity shock (Figure 2, panel (b)). Agents expect future productivity to be high, so they raise consumption and investment. However after four periods, productivity does not increase as expected. Households realize they are poorer than expected; they reduce consumption and investment, but do so slowly because of the investment adjustment costs. The marginal value of capital falls, so utilization, labor, and output all fall faster than the capital stock, which declines slowly because investment remains elevated.

3.3 Noise-Driven Fluctuations

The effects of a noise shock are plotted in Figure 2, panel (b). The economy experiences a boom and bust that looks like a typical business cycle with comovement, except without any change in productivity. Although if productivity is mismeasured by excluding utilization, it will resemble a traditional business cycle model in which measured productivity also rises. This noise-driven fluctuation occurs with or without the information friction, but the boom and bust are much larger with it.

Noise-driven fluctuation are exacerbated by the information friction. To understand this effect, I compare three information structures: (1) the baseline dispersed information model where agents only see signals on their own islands, (2) a *common information* version of the model where agents see the information sets of all islands including their news shocks but not its constituents components, and (3) a *full information* version in which agents have common information and can also observe all fundamental shocks so that they know whether productivity or noise drive news signals. I plot the macroeconomic responses to aggregate shocks with these three information structures in Figure 3.

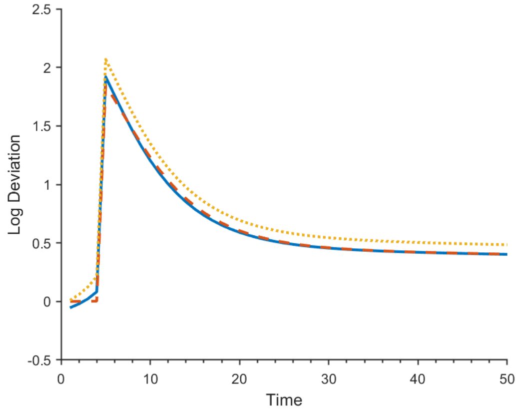
Agents are more responsive to noise shocks than if they had common information (Figure 3, panels (b) and (d)). Why? In the baseline model, the news signal (16) is the sum of three indistinguishable components: the idiosyncratic productivity shock $\epsilon_{\hat{a},i,t+\kappa}$, the aggregate productivity shock $\epsilon_{a,t+\kappa}$, and the noise shock ζ_t. While under common information, agents can identify the aggregate component of the news signal:

$$\nu_{i,t} - \epsilon_{\hat{a},i,t} = \epsilon_{a,t+\kappa} + \zeta_t \tag{19}$$

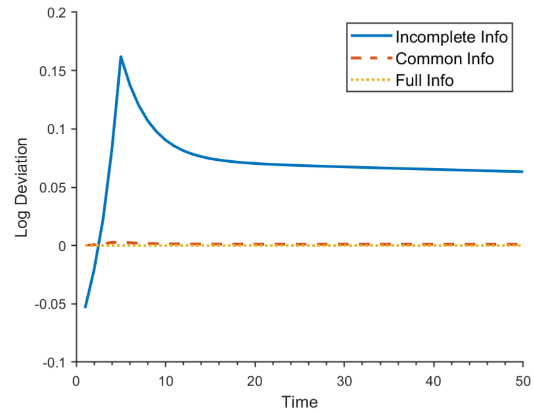
The noise shock is a relatively large component of this aggregate news signal, compared to the baseline model's news signal (16). As a result, agents are more responsive to the baseline news signal, because it is more likely to be driven by changes to productivity instead of noise. Thus aggregate variables are more elastic to noise shocks under the information friction.

Similarly, agents are more responsive to noise shocks in the common information model than in the full information model, where there are no noise-driven fluctuations. Under full information, agents see all of the shocks, so they can fully distinguish the noise from concurrent productivity shocks. Therefore they do not change any allocations in response to a noise shock (Figure 3, panels (b) and (d)). This is the first-best outcome for agents, as they have a strictly larger information set than in the other models.

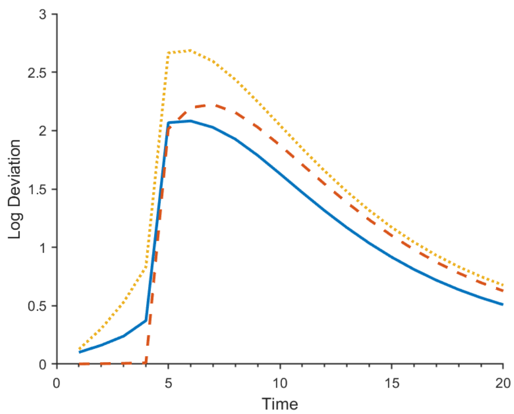
Islands cannot tell if they are in a noise-driven expansion; they have rational expectations and forecast optimally conditional on their information sets. But if policy makers have more information than islands - say, if they observe investment, consumption, or other macroeconomic aggregates - they can improve outcomes even without fully eliminating the information friction. I study this possibility in Section 4.



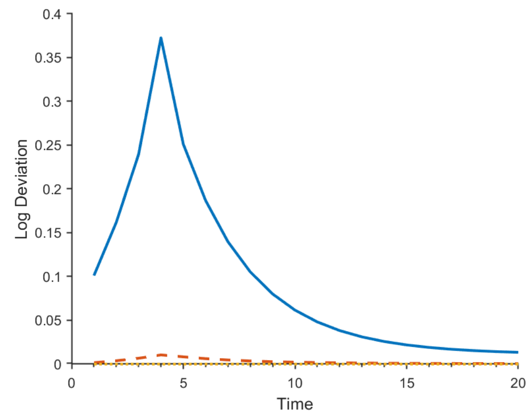
(a) Productivity Shock: Consumption Distortion



(b) Noise Shock: Consumption Distortion



(c) Productivity Shock: Output Distortion



(d) Noise Shock: Output Distortion

Figure 3: Distortions due to the Information Friction

3.4 Amplification

The information friction also amplifies the economy's response to aggregate productivity shocks (Figure 3, panels (a) and (c)) relative to the common information case.

Under common information, the news signal is a poor predictor of aggregate productivity, so households are relatively inelastic to the aggregate news signal (19), compared to the dispersed information case where they could not disentangle the aggregate news signal from their more informative idiosyncratic news. Therefore the common information households barely adjust their consumption or investment in anticipation of future aggregate productivity changes. Conversely, if households have full information, then they perfectly predict future aggregate productivity, so consumption and investment move much more in anticipation.

After the productivity shock is realized, the economy booms under all information structures. However, the more elastic investment is to the productivity shock, the larger the boom. The dispersed information economy preemptively accumulates more capital than the common information economy, so the dispersed information boom is larger once the productivity improvement arrives. The full information economy is even more elastic to aggregate productivity shocks, so it features an even larger boom.

On net, the information friction amplifies aggregate volatility relative to either of the other information structures. Table 2 documents this amplification, reporting the variance of macroeconomic aggregates relative to the first-best full information equilibrium. The dispersed information economy is more volatile because the information friction amplifies the responses to noise shocks, while generating similar responses to aggregate productivity. However the noise shocks are the largest contributor; even though noise does not produce as large of a boom as productivity (Figure 3) the noise shock has much larger variance. The contribution of the noise shock to aggregate volatility can be seen by examining the variance decomposition of how much different shocks contribute to the variances of the macroeconomic aggregates.

Information Structure	Consumption	Investment	Capital	Output
Aggregate variance relative to full information				
Dispersed Information (baseline)	166%	407%	217%	184%
Common Information	75%	67%	69%	73%
Share of variance due to noise shock				
Dispersed Information (baseline)	55.0%	84.0%	66.3%	60.0%
Common Information	$< 0.1\%$	0.2%	$< 0.1\%$	$< 0.1\%$

Table 2: Volatility Amplification

The information friction allows noise shocks to drive a large share of aggregate volatility. Table 2 documents the contribution of noise shocks under dispersed information and under common information. With common information, the noise shocks contribute nearly nothing to aggregate volatility, because agents know their aggregate news signals are noisy and thus barely respond to them. Under dispersed information, agents are much more responsive to the news signals because they are informative about idiosyncratic productivity. The noise shock's effects in the common information model is consistent with traditional findings that noise shocks in FIRE RBC models have small effects on aggregate volatility (Chahrour and Jurado, 2018). The information friction allows for noise shocks to play a much larger role in driving business cycles, consistent with reduced form empirical evidence (Chahrour and Jurado, 2022); indeed, the noise variance σ_ζ^2 is calibrated in order to deliver this result. Adjusting some parameter values can increase the contribution of the noise shocks by even more; Section 5.2 discusses these effects.

4 Optimal Policy

In this section I study policy rules by which policymakers change economy-wide investment incentives in response to aggregate shocks. If policymakers have more information than agents in the model, policy can improve outcomes. I find that the optimal policy dampens volatility by discouraging investment after a negative aggregate news signal.

Do policymakers have an informational advantage? Empirical evidence suggests that they do. Famously, Romer and Romer (2000) find that the Federal Reserve has more information about future inflation and GDP than is available to public, which allows the Fed to forecast more accurately than private forecasters. Recent work by Nakamura and Steinsson (2018), Jarociński and Karadi (2020), and Miranda-Agrippino and Ricco (2021) among others reveals that a large component of the macroeconomic response to monetary policy shocks is due to an information effect, whereby the central bank's private information is revealed to the public.

If allowed, a policymaker with superior information can simply share their information with households to improve welfare by relaxing the information friction. I do not allow them to do so. Instead, I consider second-best constrained-optimal policies where the information friction cannot directly be subverted. This is because the information friction should not be taken too literally; the literature considers these frictions to represent endogenous ignorance of some variables, as in Sims (2003) and other rational inattention theories, rather than fundamental constraints. Indeed, a large empirical literature finds that agents willfully disregard macroeconomic aggregates that are relevant for forecasting.[8] Agents choose not to employ useful information, even when policymakers share it freely. For example, much of agents' poor inflation forecasting is explained by their poor inflation *perceptions* (see e.g. Jonung (1981) or Ranyard, Missier, Bonini, Duxbury, and Summers (2008) for household evidence, and Kumar, Afrouzi, Coibion, and Gorodnichenko (2015) for firms).

One reason agents might not be aware of time series that are useful for forecasting aggregates is that they rarely have a need to form expectations over aggregate variables. It is more relevant to forecast their own prices, incomes, and circumstances, which they tend to do more accurately (Andrade, Coibion, Gautier, and Gorodnichenko, 2022). So instead I focus on policies that directly affect agents' decisions, while still allowing them to learn endogenously from the policies.

[8] See for further examples Coibion, Gorodnichenko, and Kamdar (2018), Coibion, Gorodnichenko, and Ropele (2020), Kohlhas and Walther (2021), Candia, Coibion, and Gorodnichenko (2021), or D'Acunto, Malmendier, and Weber (2023) among many others.

4.1 Policy Rules

The instrument with which policymakers influence the economy is the investment wedge $\tau_{i,t}$. The wedge has an aggregate component τ_t, which policymakers control, and an idiosyncratic component $\epsilon_{\tau,i,t}$ that is stochastic and island-specific:

$$\tau_{i,t} = \tau_t + \epsilon_{\tau,i,t}$$

The wedge $\tau_{i,t}$ modifies the Euler equation (6) by

$$Q_t = e^{\tau_{i,t}} E_{i,t} \left[\Lambda_{i,t+1} \left(R_{K,i,t+1} U_{i,t+1} + Q_{i,t+1}(1 - \delta(U_{i,t+1})) \right) \right] \tag{20}$$

The investment wedge is a reduced form stand-in for any policy that distorts aggregate savings and investment decisions. This could be direct capital taxes, or policies that affect the wedge in other ways; Chari, Kehoe, and McGrattan (2007) explore how financial frictions or other model features manifest as an investment wedge.[9] Manipulating this wedge is a useful method for improving outcomes, because it affects the allocation of goods between consumption and investment, and the consequence of the information friction is that consumers consume too much or too little in response to different shocks.

When the investment wedge $\tau_{i,t}$ is included in the model, I assume that agents observe the wedge directly. This introduces a fourth signal to an agent's information set, so an additional shock is needed to maintain the information friction. This is why I assume the wedge is affected by $\epsilon_{\tau,i,t}$, an island-specific shock, in addition to the economy-wide policy. I let this shock be purely idiosyncratic so as not to introduce an additional source of aggregate volatility. The standard deviation of this shock is set to $\sigma_\tau = 0.05$, the cross-sectional standard deviation of interest rates for European firms (Rojas, 2018).

The policies are simple linear rules that apply in perpetuity. This is akin to the Taylor-type rules commonly studied in New Keynesian models, rather than Ramsey plans that address dates of implementation and time consistency. At any point in time, a policymaker might have an incentive to lie to households and deviate from the rule; I do not allow them to do so. In order to keep the analysis tractable, I consider policies parameterized by at most two coefficients, depending on the information set of the policymaker.

If the policymaker has common information, they can observe the aggregate news signal (19): the sum of the aggregate productivity shock $\epsilon_{a,t+\kappa}$ and the aggregate

[9] As another practical example, this wedge might also represent convenience yields. If financial assets provide utility from liquidity or other properties, it introduces an investment wedge in the Euler equation corresponding to the asset, as in Krishnamurthy and Vissing-Jorgensen (2012) or Nagel (2016). Capital can either provide convenience yields itself or be repackaged into safe and risky asset tranches to take advantage of the yields as in Caballero and Farhi (2018). Monetary policymakers might then affect convenience yields by directly purchasing risky claims to capital, which the Federal Reserve did during the 2020 recession (D'Amico, Kurakula, and Lee, 2020).

noise shock ζ_t. But they cannot distinguish between the two components. In this case, the policy rule is

$$\tau_t = b_\nu \left(\sum_{k=0}^{\kappa-1} \epsilon_{a,t+\kappa-k} + \zeta_{t-k} \right) \tag{21}$$

b_ν is the coefficient on both news and noise shocks, which are co-linear to the policymaker. κ is the number of periods between when news is received and productivity is realized. The summation appears so that the policy is implemented uniformly over the entire window between news and realization.

If the policymaker has full information, they can separately identify the noise shock ζ_t from news about the aggregate productivity shock $\epsilon_{a,t+\kappa}$. In this case, the policy rule is

$$\tau_t = b_a \left(\sum_{k=0}^{\kappa-1} \epsilon_{a,t+\kappa-k} \right) + b_\zeta \left(\sum_{k=0}^{\kappa-1} \zeta_{t-k} \right) \tag{22}$$

With full information, policymakers can respond to news and noise shocks differently, so the policy rule has two parameters: b_a is the coefficient on aggregate productivity shocks, and b_ζ is the coefficient on noise shocks.

I also consider an information structure where policymakers have no information about the households' noise shocks. Instead, policymakers receive their own noisy signal ψ_t of aggregate productivity:

$$\psi_t = \epsilon_{a,t+\kappa} + \zeta_t^G \tag{23}$$

where the noisy shock ζ_t^G is i.i.d. with the same variance as the households' noise shock ζ_t. Thus the policymaker's noisy signal ψ_t is as informative about productivity as the common information signal ν_t. One benefit of this approach is that the idiosyncratic shock to the intertemporal wedge $\epsilon_{\tau,i,t}$ is no longer necessary to maintain the dispersed information structure. Therefore, when studying this information structure, I set $Var(\epsilon_{\tau,i,t}) = 0$. Introducing this additional aggregate shock that is not directly observed by households implies that even when agents observe the aggregate policy instrument τ_t directly, neither common nor full information are revealed. There are still more shocks than signals, and different islands make different forecasts. However, the policymaker no longer has an informational advantage; it reveals its private information through the policy instrument.

4.2 Optimal Policy

The policymaker chooses a rule that maximizes expected welfare. I follow a standard approach (Rotemberg and Woodford, 1997): policymakers maximize the unconditional expected discounted utility of a household, i.e. the unconditional expectation of equation (1) over time and households. Appendix B details how this welfare objective is calculated, including how to account for the effects of policy on the stochastic steady state. This approach is useful for tractability, as it avoids issues of time inconsistency that typically arise in policymaking problems.

To find the optimal policy, I search for the policy parameters that maximize the welfare objective. When policymakers have common information, this is a one-dimensional search for b_ν, the coefficient on the aggregate news signal. When policymakers have full information, it is a two-dimensional search for b_a and b_ζ, the coefficients on aggregate productivity and noise respectively. This second case allows for policy outcomes that are at least as good as when policymakers are restricted to common information, which is nested as a special case when $b_a = b_\zeta$.

Table 3 reports the optimal policy parameters for each information structure.

Policy Rule	Parameter values	Consumption-equivalent improvement
No policy	$b_\nu = 0,\ b_a = 0,\ b_\zeta = 0,\ b_\psi = 0$	0%
Common information optimum	$b_\nu = -0.13,\ b_a = 0,\ b_\zeta = 0,\ b_\psi = 0$	1.6%
Full information optimum	$b_\nu = 0,\ b_a = 0.24,\ b_\zeta = -0.14,\ b_\psi = 0$	2.5%
Noisy signal optimum	$b_\nu = 0,\ b_a = 0,\ b_\zeta = 0\ ,\ b_\psi = 0.004$	$< 0.01\%$

Table 3: Optimal Policy

Consumption-equivalent improvements are the welfare improvements relative to the no-policy baseline, expressed as percentages of the deterministic steady state consumption level. Under "common information", the policymaker sees the average noisy signal ν_t observed by households. The "full information" policymaker sees the true shock processes for a_t and ζ_t. The "noisy signal optimum" refers to policymakers who do not access household information, but instead observe a noisy signal ψ_t of aggregate productivity. Policy parameters are elasticities of the investment wedge to these signals.

When policymakers have common information, their best policy is to "lean against the wind." The optimal coefficient on aggregate news is $b_\nu = -0.13$, which says that policymakers should discourage investment after a positive news signal. Absent any policy, agents increase investment after a news shock, so the optimal policy dampens the aggregate response. This is because, from the perspective of the informed policymakers, islands are responding "too much" to their news signals, which conflate informative signals about idiosyncratic productivity with noisy signals about aggregate productivity. Adopting this policy cuts the consumption standard error by more than a half.

When policymakers have full information, they can respond differently to aggregate productivity and noise shocks. In response to an aggregate productivity shock, the optimal policy amplifies agents' response: $b_a = 0.24$ says that when agents increase investment after a positive news shock, the policy should induce them to increase investment even further. Conversely, the optimal policy dampens agents' response to an aggregate noise shock: $b_\zeta = -0.14$ says that when agents increase investment after a positive aggregate noise shock, policy should encourage them to invest less. Adopting this two-parameter policy raises consumption-equivalent welfare by 2.5% of steady-state consumption, roughly one and a half times as much as the restricted common information policy.[10] Still, these values are small in absolute magnitude because the welfare costs of business cycles are famously small when measured as aggregate consumption equivalents (Lucas, 1987).

When policy makers observe their own noisy signal of productivity ψ_t instead of seeing the households' noise, their optimal policy is relatively muted. Their goal is still to boost investment after receiving positive news about future productivity, but the policy parameter b_ψ is small. Policymakers are inelastic to their signal, because it is noisy and they risk introducing additional non-fundamental volatility to the macroeconomy by mistakenly responding to their noise shocks. But most importantly, their noise is uncorrelated with the households' information. In contrast, what was useful about the common information signal ν_t is that it was informative about the noise that affected households ζ_t, even though it was an equally poor signal of productivity. But why is b_ψ not even smaller? Any non-zero value reveals the policymaker's information, but simply sharing information is not optimal when there are more shocks than signals. The policymaker still has an incentive to correct the inefficiency caused by households placing too little weight on their own signals when forecasting (Morris and Shin, 2002).[11]

The common pattern is that optimal policy dampens the investment response to noise shocks whenever possible. Under both common and full information structures, policymakers are able to more accurately now-cast the noise shock than individual islands. It is wasteful for islands to raise investment after a noise shock, so when policymakers expect that there was an aggregate noise shock, they dampen islands' investment response.

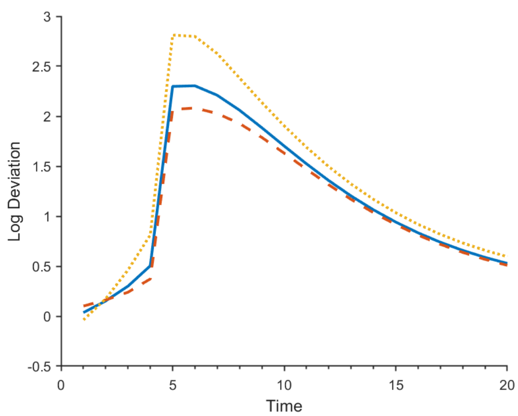
Figure 4 demonstrates how the optimal policy dampens the output response to news. The solid blue line plots the impulse response of aggregate output when there is no policy intervention. If the policymaker has common information (the dashed red line), they discourage investment when there is a positive news signal. Whether the news signal is driven by future productivity (panel (a)) or by a noise shock (panel

[10]It is difficult to improve further with a policy rule as simple as (21) or (22). In principle, a fully dynamic, island-specific policy rule might be able to recover the first-best full information equilibrium. But such a policy rule would be impossibly complicated. After all, the island's equilibrium choices are infinite dimensional objects: irrational lag operator polynomials of current and past shocks (Huo and Takayama, 2016). Instead, the simple policy rule is not flexible enough to fully counteract the effects that optimism about far-future productivity has on concurrent consumption.

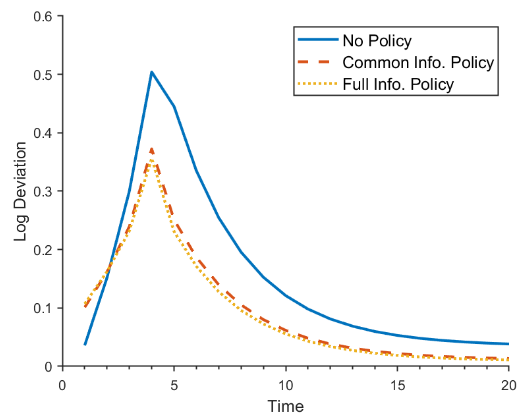
[11]Another force helps keep b_ψ small. The more precise the noisy signal, the closer the economy moves to the first-best when agents learn the policymaker's information, and the less a policymaker can gain by distorting the investment margin. In the extreme case where the policymaker's signal has $Var(\zeta_t^G) = 0$, then it would be optimal to choose b_ψ arbitrarily small, so that the policy instrument would reveal the productivity shock exactly and households could infer the values of all shocks.

(b)) the output IRF falls below the IRF corresponding to no policy. If a noise shock drove the news signal, then the resulting macroeconomic fluctuation is much smaller thanks to the policy. But there is a trade-off: if a productivity shock drove the news signal, then there is too little investment and the economy does not boom enough.

However in the full information case (the dotted yellow line) the policymaker has more flexibility. They can do even better by implementing different changes to the investment wedge depending on whether the driving shock is news or noise. As before, they dampen the investment response to a noise shock, so output booms by less (panel (b)). But they also magnify the investment response if the news is driven by future productivity, so output booms by more (panel (a)). This is because agents know that their news signal might be caused by a noise shock, and do not respond as much as they would if it accurately predicted productivity. A policymaker who knows if news is driven by productivity can make agents better off by amplifying their response.



(a) Aggregate Productivity Shock



(b) Aggregate Noise Shock

Figure 4: Policy Effects on Output Impulse Response Functions

The parameter signs are the most meaningful results in Table 3. The actual parameter values are more challenging to draw conclusions from. For example, $b_\nu = -0.13$ is a small elasticity, but that's because the GHH preferences are close to log utility, which makes their consumption-saving decision sensitive to the interest rate. It is more useful to examine the quantitative changes to household behavior, where the effects are large. In the case where the policymakers have common information, the optimal policy reduces the immediate investment elasticity to aggregate news from 0.5 to 0.2 (Figure 4). And when policymakers have full information, the optimal policy reduces the immediate consumption elasticity to noise shocks by a similar magnitude. The relevant conclusion is not that the investment wedge should have exactly a -0.13 elasticity to aggregate news, but rather that the policy should significantly moderate the investment response in order to dampen the macroeconomic fluctuation caused by a noise shock.

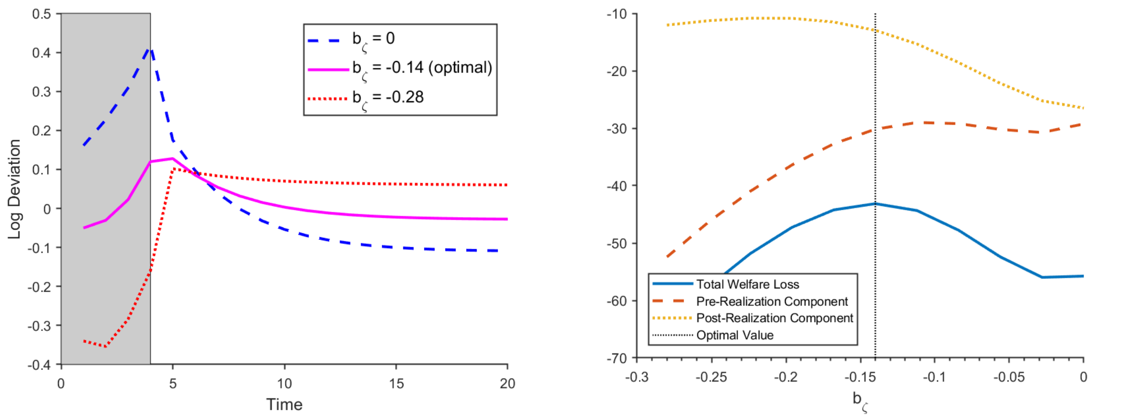
The qualitative conclusion that policy should "lean against the wind" with regards to noise echoes Dupor (2005). But in contrast to noise shocks, Dupor studies the optimal policy response to "expectations shocks" which are reduced form behavioral distortions to beliefs about future productivity. The policy instrument is money growth, which has real effects in a monetary model with Rotemberg (1982) pricing and capital adjustment costs. Households have standard preferences, so without a policy intervention, the expectations shock causes a large increase in output, investment, labor, and real interest rates, and a large decrease in consumption. The monetary authority perfectly controls inflation, so it affects the real economy by manipulating the labor wedge. Dupor finds that the optimal response to expectations shocks is to prevent the capital expansion by driving up the labor wedge to create a recession. Thus, whether the average forecasting error is exogenous or microfounded, and whether the policy instrument is a labor or investment wedge, the conclusion is that policymakers should counteract the investment boom.

4.3 Policy Trade-offs

What trade-offs does a policymaker face when setting their policy rule for the investment wedge? The benefits and costs of the policy are intertemporal: the more aggressively a policymaker moderates the effects of noise in the short run, the worse misallocation they create in the long-run.

To illustrate this trade-off, it is useful to examine how different policy rules affect the dynamics of household marginal utility. The policymaker's objective is to maximize expected household welfare, and one component of the welfare function is the variance of the stochastic discount factor. This component of welfare is maximized by smoothing household marginal utility over time. The investment wedge is a useful tool for accomplishing this aim - the policymaker can directly affect expected marginal utility growth through the Euler equation. But its usefulness has limits: the wedge can easily distort substitution effects, but struggles to control income effects. This limitation creates a trade-off. Smoothing marginal utility in the short run leads to too much or too little capital accumulation, so that after the signal horizon κ expires and households learn about productivity, there is a large consumption adjustment.

Figure 5 demonstrates the trade-off. Panel (a) plots the impulse response of average marginal utility to the noise shock ζ_t under several policy rules. In each case, the policymaker has full information, and adopts the productivity elasticity from the optimal policy rule ($b_a = 0.24$), but a different elasticity to the noise shock b_ζ. When the policy is passive ($b_\zeta = 0$, the dashed blue line), agents respond as described in Section 3: after a noise shock, they expect high future productivity, so they increase investment, and pay for it by decreasing consumption. Thus the shock increases marginal utility, but after no productivity improvement is realized, agents consume out of their accumulated capital, and marginal utility falls below the steady state for many periods. This impulse response function is relatively smooth before the signal



(a) Marginal Utility IRF to a Noise Shock (b) Effects of Policy on Welfare Components

Figure 5: Intertemporal Policy Trade-offs

horizon expires (the shaded gray region). But it is not very smooth afterwards; the variance of marginal utility growth is large. The policymaker can do better.

When the investment wedge is especially elastic to the noise shock ($b_\zeta = -0.28$, the dotted red line), the policymaker aggressively discourages investment, so marginal utility falls after the shock, then jumps up when no productivity improvement is realized. The *benefit* of this aggressive policy is that marginal utility growth is effectively smoothed after the signal horizon. This is because capital is well below the steady state and adjustment is costly, so the economy grows gradually. However the *cost* of this aggressive policy is that marginal utility is not smoothed at all in the initial periods (the shaded gray region). The optimal policy (solid purple line) balances these trade-offs so that marginal utility is relatively smooth over its entire response.

Of course, there is more to welfare than the variance of the average stochastic discount factor. A decomposition of the welfare function quantifies the general trade-off between the pre- and post-realization periods. Specifically, the welfare calculation is a function of variances and covariances of the endogenous variables. Covariances are linear combinations of pre- and post-realization terms, so expected welfare can also be decomposed into pre- and post-realization components:[12]

$$E\left[W_{i,t}\right] = W_{Pre} + W_{Post}$$

W_{Pre} captures the component of the welfare function that is due to the variances and covariances in the first κ periods after shocks. The residual component W_{Post} includes all of the effects after the κ-period horizon, when signals either realize as productivity or noise.

The main policy trade-off is between the W_{Pre} and W_{Post} components of expected welfare. Figure 5 panel (b) illustrates the trade-off by plotting the value of each com-

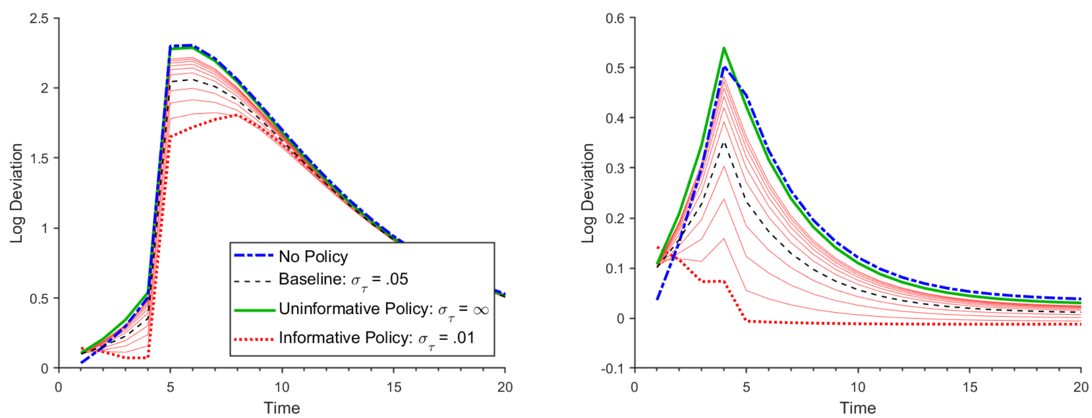
[12]Appendix B.3 derives this decomposition.

ponent for a range of policy parameters. The expected welfare is negative, because it is calculated relative to the FIRE optimum, as described in Appendix B. When policymakers are passive in response to a noise shock ($b_\zeta = 0$), the post-realization component (dotted yellow line) is most negative; this is consistent with the poor marginal utility smoothing outside of the gray region in panel (a). When policymakers are aggressive in response to a noise shock ($b_\zeta = -0.28$), the pre-realization component (dashed red line) is most negative; this is consistent with the poor marginal utility smoothing inside the gray region in panel (a).

The policymaker would like to balance this trade-off. The policy is optimal when the marginal benefit of increasing W_{Pre} equals the marginal cost of decreasing W_{Post}. This optimal value (dotted black line at $b_\zeta = -0.14$) maximizes the total expected welfare (solid blue line).

4.4 Incentives or Information?

There are two channels through which the policy affects individuals. The first is the incentive channel: when the intertemporal wedge increases, it distorts the incentives for individuals to save or consume. This is the main mechanism that I have discussed thus far. However there is a second channel: changes to the intertemporal wedge contain information about the aggregate state of the economy, because policymakers have an informational advantage.

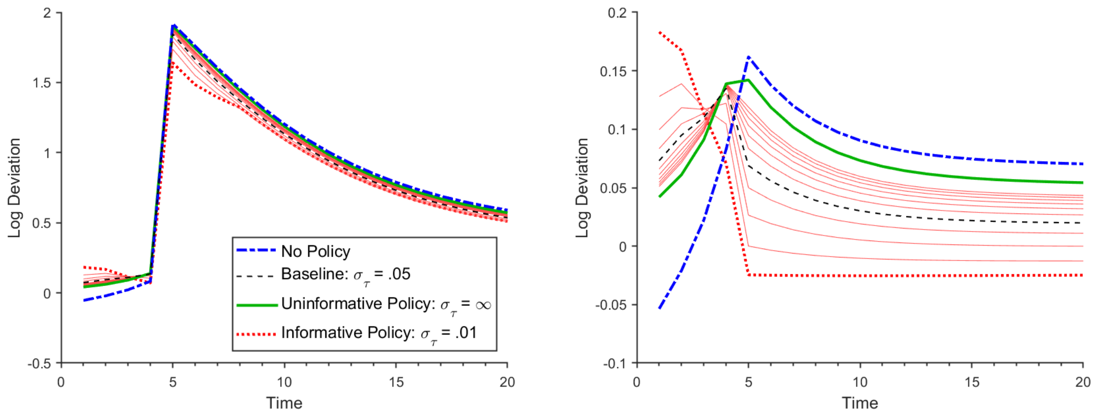


(a) Aggregate Productivity Shock

(b) Aggregate Noise Shock

Figure 6: Effects of Policy Information Content on Output Impulse Responses

What is the relative importance of these channels? To explore this question, I examine how the dynamics are affected when I change the informational content of the intertemporal wedge. The wedge $\tau_{i,t}$ is determined by both aggregate policy and an idiosyncratic shock $\epsilon_{\tau,i,t}$ (21). Therefore the idiosyncratic shock variance σ_τ^2 controls how informative the wedge is about the aggregate policy choice, and thus the aggregate state of the economy. For example, when σ_τ^2 is large, the wedge $\tau_{i,t}$ has low correlation with aggregate news, and thus little power for predicting productivity. In contrast, when σ_τ^2 is small, the wedge is highly correlated with aggregate news due to the policy rule, so observing the wedge affects individuals' forecasts and behavior.



(a) Aggregate Productivity Shock (b) Aggregate Noise Shock

Figure 7: Effects of Policy Information Content on Consumption Impulse Responses

Figure 6 plots how changing the information content of the intertemporal wedge affects GDP dynamics. The blue line is the IRF for output in the baseline model without any policy. All other lines plot economies where policymakers have common information, with the optimal policy parameter b_ν set as in Table 3. The solid green line is the case where the information channel is shut down: σ_τ^2 is large and households infer nothing about the aggregate state from their intertemporal wedge.[13] The thick dashed red line is the opposite: policy is very informative because the intertemporal wedge nearly reveals the aggregate news signal.

Regardless of how informative it is, the policy serves to raise output in the short run in order to lower it in the medium run. When aggregate productivity (panel (a)) or noise (panel (b)) increase, policymakers see an aggregate news signal (19) and increase the average intertemporal wedge, encouraging consumption on impact (Figure 7). To afford the additional consumption, islands raise output and reduce investment. So after productivity is or is not realized, the economy has less capital and returns to steady state faster.

When the information channel is shut down, policy has a muted effect (the solid green line in Figures 6 and 7). Households only increase consumption by a modest amount relative to output, so investment remains elevated and capital accumulates, prolonging the boom and bust.

[13] The infinite variance of the idiosyncratic wedge should not be interpreted literally - this would imply an infinite variance of asset prices, per equation (20). Rather, this is the outcome when households are assumed to use no information from the intertemporal wedge when forecasting.

However, when the policy contains information about the aggregate economy, the effects on output and consumption are magnified. When households see the investment wedge rise, they recognize that their observed news was more likely to be driven by aggregate noise shocks. So they have less incentive to invest, and instead raise current consumption even further.

In all cases, the policy encourages consumption and discourages investment after positive aggregate news. When the information channel is larger, households are more elastic to the policy instrument, and their response moves in the same direction relative to the case without policy. In the baseline calibration (the dashed black line) the information effect appears large relative to the incentive effect. But that does not imply that incentives are unimportant, because these comparisons keep the policy parameter constant. Rather, if the policy were less informative, the optimal policy elasticity would be much larger, in order to effect similar moderation of the noise-driven fluctuation.

5 Sensitivity Analysis

This section studies two sensitivity analyses. First, I allow noise shocks to have an idiosyncratic component, and vary its contribution to the news signal. Second, I study how business cycle amplification is affected varying the calibrated parameters.

5.1 Idiosyncratic Noise

In the baseline model, noise was a purely aggregate shock, which was crucial for the amplification of noise shocks. In this section, I relax that assumption. Now, the possibility result becomes: if agents' noise shocks are *sufficiently correlated* then noise shocks can amplify business cycles.

The noise shock is modified to have an idiosyncratic component. Agents now receive the signal

$$\nu_{i,t} = \epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa} + \zeta_{i,t} \tag{24}$$

which modifies equation (16) so that $\zeta_{i,t}$ is an island-specific noise component. This component is the sum of two shocks:

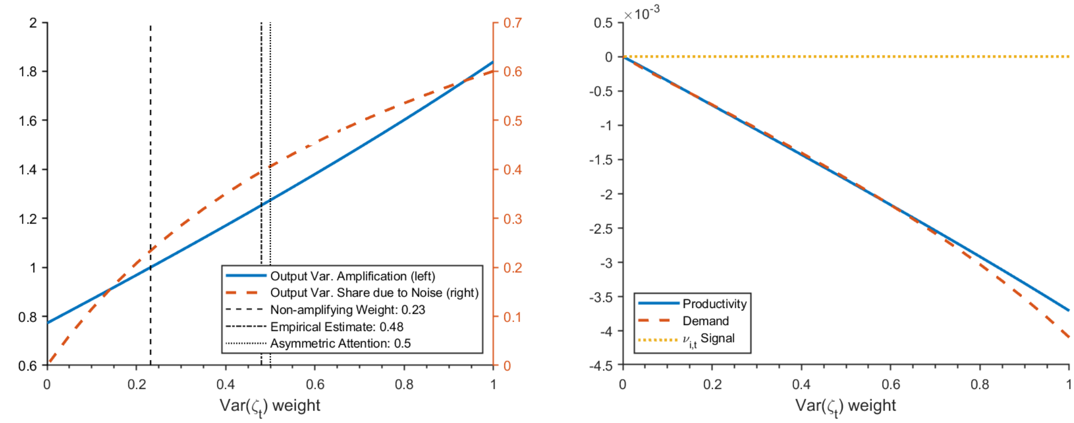
$$\zeta_{i,t} = \zeta_t + \hat{\zeta}_{i,t}$$

$\hat{\zeta}_{i,t}$ is an i.i.d. noise shock that is specific to island i and mean zero in the population. As before, ζ_t is an aggregate noise shock.

I analyze the effects of idiosyncratic noise in the following way. The noise variance is set to $Var(\zeta_{i,t}) = (0.115)^2$ as in the baseline calibration. Then, the shock variances are determined by a single weighting parameter w_ζ such that:

$$Var(\zeta_{i,t}) = w_\zeta Var(\zeta_t) + (1 - w_\zeta) Var(\hat{\zeta}_{i,t}) \tag{25}$$

The weight w_ζ controls how much of noise is aggregate versus idiosyncratic. When $w_\zeta = 1$, noise is only due to the aggregate shock ζ_t, recovering the baseline model. When $w_\zeta = 0$, noise is only due to the idiosyncratic shock $\hat{\zeta}_{i,t}$. Keeping $Var(\zeta_{i,t})$ fixed while changing the w_ζ is a useful exercise, because it keeps the exogenous signal's information structure of households unaffected except for the correlation of noise across islands. I calibrate the weight w_ζ in two ways: a theoretical model of rational inattention, and an empirical approach using industry-level data.



(a) Amplification with Idiosyncratic Noise (b) Relative Forecast Error Variances

Figure 8: Effects of Idiosyncratic Noise

Figure 8 panel (a) plots how amplification depends on the aggregate noise weight w_ζ. The "Output Variance Amplification" (left axis) captures how much the information friction amplifies aggregate volatility by measuring the aggregate output variance under dispersed information relative to full information. Meanwhile, the "Output Variance due to Noise" (right axis) captures how much the noise contributes to business cycle volatility by measuring what share of the aggregate output variance is attributable to aggregate noise shocks.

It is clear that correlated noise is crucial for the baseline model to achieve amplification. As the weight w_ζ goes down, amplification falls and the share of output volatility drive by noise shocks declines. When less than a quarter of noise is due to aggregate shocks (the "Non-amplifying weight" in Figure 8) the model no longer features amplification. This non-amplifying weight is still relatively large, implying that the variance of idiosyncratic noise is only three times as large as aggregate noise, while the variance of idiosyncratic productivity is more than an order of magnitude larger than aggregate productivity. Finally, as the weight goes to zero, noise shocks are entirely idiosyncratic, so noise no longer appears in the output variance decomposition; in this case, the effect of noise on business cycles is only indirect, by distorting agents' forecasting ability.

What is a reasonable value for the weight w_ζ? The dotted line labeled "Asymmetric Attention" in Figure 8 panel (a) plots the calibrated weight that solves an optimal attention problem. Agents control both noise variances and can pay a cost to reduce the variance of either noise shock. Appendix C describes the assumptions of this model in detail, but one conclusion is clear: agents prefer their signals to place more weight on the aggregate noise shock ζ_t rather than the idiosyncratic noise shock $\hat{\zeta}_{i,t}$. This is because aggregate demand for an island's output also depends on ζ_t. Accordingly, when the aggregate weight w_ζ is higher, forecasting is improved in two ways: the noisy signal allows agents to forecast demand more accurately, and the observed demand allows agents to better disentangle noise from news in their individual signal and forecast productivity more accurately. Figure 8 panel (b) demonstrates these effects by plotting how the variance of agents' forecast errors depend on the weight w_ζ. As w_ζ increases and agents' noise is more correlated, the variance of their forecast errors for both productivity and demand decline, but their ability to forecast future noisy signals is unaffected.

This resembles an *asymmetric attention* problem (Kohlhas and Walther, 2021): different components of the signals asymmetrically affect forecasting ability, incentivizing attention to be allocated asymmetrically. Agents are only indifferent between aggregate and idiosyncratic noise in the case where $w_\zeta = 0$, because if no other islands' signals have aggregate noise, then increasing or decreasing the aggregate weight in a single agent's signal has no affect on their forecasting ability. Otherwise, agents prefer their noise to be aggregate, but how much weight they choose to place on the aggregate component depends on the cost of doing so. Appendix C.2 shows that when the cost is based on signal entropy, marginal cost is symmetric and agents will choose $w_\zeta = 1$. But if the marginal cost of reducing the a component's variance is increasing, then they will choose an interior solution where $w_\zeta \geq 0.5$, well above the non-amplifying value. Appendix C.3 presents a conservative example of this kind, where costs increase quickly enough that the optimal weight is close to one half; this is the "Asymmetric Attention" value in Figure 8 panel (a).

I also conduct an empirical calibration of the weight w_ζ. To do so, I apply the Chahrour and Jurado (2022) noise shock estimation to industry-level data. This procedure identifies noise shocks by assuming that TFP is an exogenous process. Then, if VAR-implied expectations of future TFP are not explained by current and past TFP, they must contain noisy news signals about future realizations. Noise is estimated as the component of these expectations that are orthogonal to TFP at all future, current, and past horizons. Appendix D describes this procedure in greater detail. To map these estimates to the model, I calculate the variances of aggregate and orthogonalized industry-level noise shocks, and then scale both variances so that they sum to the baseline noise variance. Converting to a weight per equation (25) gives $w_\zeta = 0.48$ in the preferred specification (the "Empirical Estimate" in Figure 8 panel (a)), but other estimates range from $0.30 - 0.67$.

Altogether, this analysis of idiosyncratic noise demonstrates that the model's amplification result is a conditional one: *if* noise is sufficiently correlated, *then* it can amplify business cycles.[14] It is not surprising that this result requires a specific condition, given that the theoretical business cycle literature has thus far found small roles for noise. The empirical literature finds large effects of aggregate noise, so correlated noise shocks provide a possible resolution to this disagreement. Both theoretical microfoundations and evidence from industry data suggest that this resolution is feasible.

5.2 Parameter Sensitivity

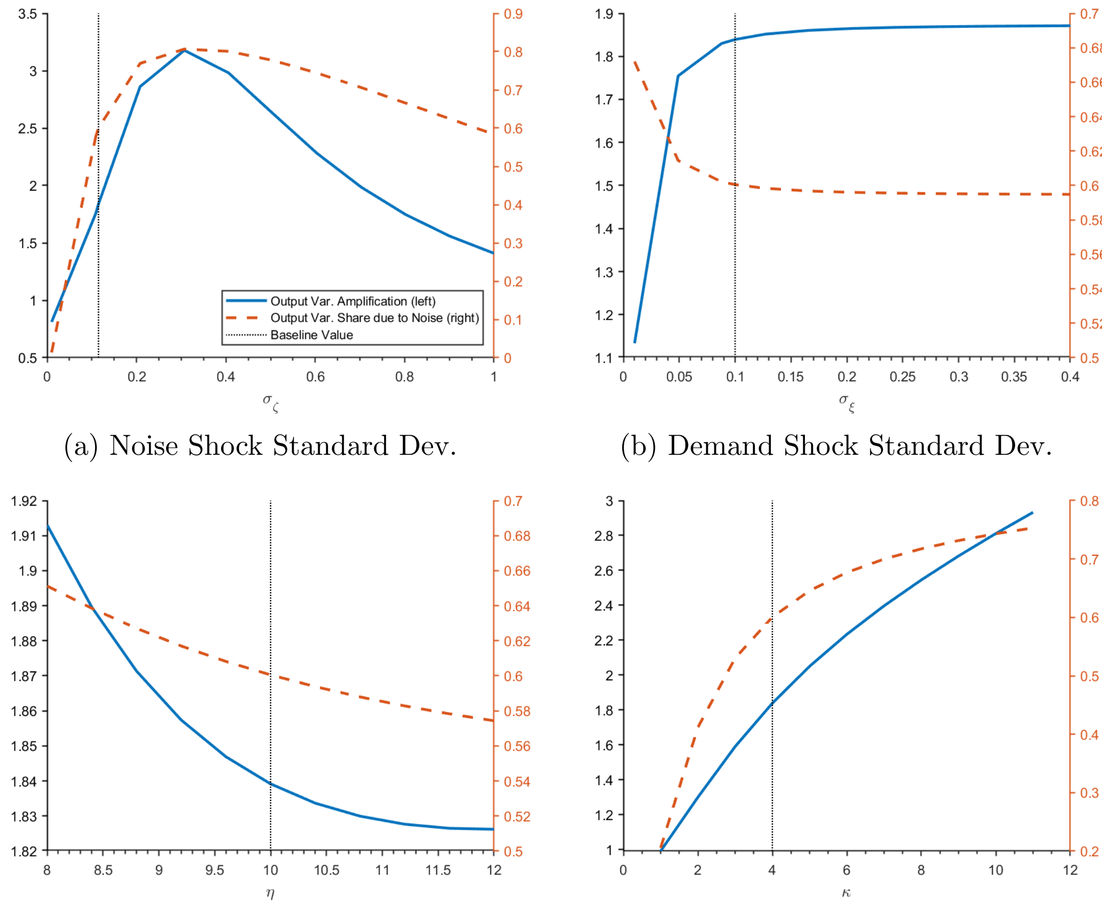
In this section, I vary the parameter values in the baseline model that are not well disciplined by the literature, and examine how changing these values affects the incidence of noise shocks and the information friction. Figure 9 plots how changing each parameter affects two quantities in the model, while keeping the remaining parameter values at their baseline levels without any policy intervention. As in Figure 8, the "Output Variance Amplification" is plotted on the left axes, measuring the output volatility relative to full information, while the 'Output Variance due to Noise" is plotted on the right axes, reporting the variance decomposition due to aggregate noise shocks.

There are three main conclusions from this analysis. First, varying the noise shock standard deviation σ_ζ is flexible enough to match almost any role for noise shocks in aggregate volatility. In the calibration, this is done to target 60%, the estimate by Chahrour and Jurado (2022), in line with other reduced form evidence, but much larger than implied by FIRE business cycle theories. Second, this ability is not constrained by the other economic parameters; varying the demand shock standard deviation σ_ξ and the elasticity of substitution η do not substantially the importance of noise for the business cycle. However, the third conclusion is that other properties of the information process are relevant. In particular, if the time between news and shock realization is short, it limits the potential influence of noise on macroeconomy volatility.

The standard deviation of the noise shock σ_ζ has a non-monotonic effect on both measures (panel (a)). When the standard deviation is small, the noise shock can only contribute a small amount to aggregate variances, even though the effects of a noise shock are large. And when the standard deviation is large, the effects of a noise shock eventually start to decline because the news signal starts to become a poor predictor of future productivity, so agents become inelastic to it. How in the intermediate region, the effects of the noise shock can be very large; indeed, no parameter can be varied to induce larger aggregate volatility than the noise shock. The baseline parameter is an intermediate value.

The standard deviation of the demand shock σ_ξ increases both measures, before

[14] However, the presence of idiosyncratic noise does not affect the qualitative conclusions about optimal policy. Appendix E demonstrates that for any $w_\zeta > 0$, policymakers are incentivized to enact countervailing policy that discourages investment after aggregate noise shocks.



(a) Noise Shock Standard Dev.

(b) Demand Shock Standard Dev.

(c) Elasticity of Substitution

(d) News Horizon

Figure 9: Parameter Sensitivity

tapering off (panel (b)). When this standard deviation is low, demand is an informative signal of aggregate output, which helps agents tell whether their news signals are driven by idiosyncratic or aggregate shocks. This in turn helps them tell whether their signal is determined by fundamentals or noise, because noise only affects the aggregate component. Thus when demand is informative, agents can better predict the aggregate state of the economy, and volatility is lower. However, there is also a counterintuitive effect: when demand is more informative, the contribution of noise shocks to the aggregate business cycle is larger. This is because islands place more weight on the demand signal than their news signal, and noise shocks contribute to a larger share of the volatility of the demand signal than the news signal. In both cases, the effect of σ_ξ plateaus rapidly, because once the variance is large, additional increases have little effect on the informativeness of the signal.

The elasticity of substitution η decreases both measures (panel (c)). When the elasticity of substitution is larger, productivity matters more for an island's revenue:

when $\eta = 1$ productivity has no effect on revenue, but as $\eta \to \infty$ productivity increases revenue one-for-one. So as η gets larger, islands care more about forecasting their own idiosyncratic productivity and less about forecasting aggregate demand. This reduces the weight they put on their demand signal, which is disproportionately driven by aggregate noise shocks, reducing the effects of noise on business cycle volatility.

Finally, panel (d) plots the news horizon κ, which is the number of periods between when a productivity shock affects the news signal and when the change in productivity is realized. Lengthening the horizon increases the amount of time agents have to act on a news signal, expecting future productivity improvements. This gives them more time to ramp up investment, raising the capital stock and future output. Unlike the other parameters, this does not clearly affect the immediate elasticity of investment to news signals, but it increases the duration of noise-driven expansions before they collapse, which raises aggregate volatility and the contribution of noise shocks.

6 Conclusion

In this paper I studied the business cycle effects of noise shocks when agents have dispersed information. I found that the information friction allows noise shocks to produce large macroeconomic fluctuations that are common enough to generate a large share of business cycle volatility (Table 2). The crucial assumption is that individuals' noise shocks are sufficiently correlated (Figure 8).

Then, I examined optimal policy rules for an intervention via the investment wedge. I found that policymakers should discourage investment during booms and encourage it during busts. This conclusion holds regardless of what the policymaker's information set is.

To implement such a policy, it is essential that policymakers work to identify aggregate news in the economy – or even better, aggregate noise. If they have an informational advantage that allows even partial identification of such objects, moderation of noise-driven fluctuation becomes feasible.

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A Linearization

Solving the model requires log-linearizing the equilibrium conditions. I report these linear equations in this section. There are exactly as many equilibrium conditions as endogenous choice variables. Some additional variables may be endogenous but individual agents take as exogenous (such as aggregate demand).

Steady state values are denoted with overbars (e.g. steady state consumption is $\bar{C}$) while log deviations are denoted as lower case variables (e.g. the consumption deviation is $c_{i,t}$).

I express the linear equilibrium conditions for the model as a set of 8 equations with 8 choice variables: output y_t^i, consumption $c_{i,t}$, capital $k_{i,t}$, investment $i_{i,t}$, labor $n_{i,t}$, utilization $u_{i,t}$, Tobin's Q $q_{i,t}$, and the stochastic discount factor $m_{i,t}$. 3 additional variables that individual islands cannot affect are: productivity $a_{i,t}$, demand $d_{i,t}$, and (when considering policy) the investment wedge $\tau_{i,t}$.

The log-linearized production function is

$$y_t^i = a_{i,t} + \alpha(u_{i,t} + k_{i,t}) + (1-\alpha)n_{i,t} \tag{26}$$

The log-linearized Euler equation is

$$q_{i,t} = \tau_{i,t} + E_{i,t}\left[m_{i,t+1} + \beta\bar{R}\left(\frac{\eta-1}{\eta}y_{t+1}^i - k_{i,t+1} + d_{i,t+1}\right) + \beta(1-\delta(\bar{U}))(q_{i,t+1} - \delta'(\bar{U})\bar{U}u_{i,t+1})\right] \tag{27}$$

where $\bar{R} \equiv 1/\beta + \delta - 1$.

The log-linearized stochastic discount factor is

$$m_{i,t+1} = \frac{1}{\bar{C}-\chi}\left(\bar{C}(c_{i,t} - c_{i,t+1}) - \chi\theta(n_{i,t} - n_{i,t+1})\right) \tag{28}$$

with χ chosen to normalize $\bar{N} = 1$.

The log-linearized first order condition for investment is

$$i_{i,t} = \frac{1}{\varphi''(1)(1+\beta)}q_{i,t} + \frac{1}{1+\beta}i_{i,t-1} + \frac{\beta}{1+\beta}E_{i,t}\left[i_{i,t+1}\right] \tag{29}$$

The log-linearized first order condition for capital utilization is

$$\frac{\eta-1}{\eta}y_t^i - u_{i,t} + d_{i,t} = q_{i,t} + k_{i,t} + \delta''(\bar{U})\bar{U}u_{i,t} \tag{30}$$

The log-linearized first order condition for labor is

$$\frac{\eta-1}{\eta}y_t^i - n_{i,t} + d_{i,t} = (\theta-1)n_{i,t} \tag{31}$$

The log-linearized law of motion is

$$k_{i,t+1} = (1-\delta(\bar{U}))k_{i,t} + \delta(\bar{U})i_{i,t} - \delta'(\bar{U})\bar{U}u_{i,t} \tag{32}$$

The log-linearized budget constraint is

$$\bar{Y}\left(d_{i,t} + \frac{\eta-1}{\eta}y_t^i\right) = \bar{C}c_{i,t} + \bar{I}i_{i,t} \tag{33}$$

B Calculating Welfare

The policymaker chooses its policy rule to maximize the unconditional expectation of household welfare. The standard approach in linear models is to evaluate the second-order approximations of the welfare function (Woodford (2003)). However, a complication arises: the deterministic steady state is not an accurate approximation point. Policy can change dynamics enough to affect the stochastic steady state.[15] The welfare derivation proceeds in two parts: first Section B.1 derives how dynamics affect the second-order terms in the welfare approximation, then Section B.2 derives how to calculate the effect of dynamics on the stochastic steady state.

B.1 Derivation of Welfare Approximation

The welfare measure $W_{i,t}$ for island i at time t is expected utility (1):

$$W_{i,t} = E_{i,t}\left[\sum_{s=0}^{\infty}\beta^s \ln\left(C_{i,t+s} - \chi N_{i,t+s}^{\theta}\right)\right] \tag{34}$$

The second order approximation around the stochastic steady state $\overline{W}$ is

$$\begin{aligned} W_{i,t} \approx \overline{W} + E_{i,t}\Bigg[\sum_{s=0}^{\infty}\beta^s \Bigg(& \frac{\overline{C}}{\overline{C}-\chi\overline{N}^{\theta}} c_{i,t+s} - \frac{\chi\theta\overline{N}^{\theta}}{\overline{C}-\chi\overline{N}^{\theta}} n_{i,t+s} \\ & + \frac{1}{2}\left(\frac{\overline{C}}{\overline{C}-\chi\overline{N}^{\theta}} - \frac{\overline{C}^2}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2}\right) c_{i,t+s}^2 \\ & - \frac{1}{2}\left(\frac{\chi\theta^2\overline{N}^{\theta}}{\overline{C}-\chi\overline{N}^{\theta}} + \frac{\left(\chi\theta\overline{N}^{\theta}\right)^2}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2}\right) n_{i,t+s}^2 \\ & + \frac{\overline{C}\chi\theta\overline{N}^{\theta}}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2} c_{i,t+s} n_{i,t+s}\Bigg)\Bigg] \end{aligned} \tag{35}$$

Unconditional expectations of the first order terms are zero, so the expected welfare deviation from the steady state is given by the unconditional expectation of the second order terms:

$$E\left[W_{i,t} - \overline{W}\right] \approx E\left[-\varpi_{CC} c_{i,t+s}^2 - \varpi_{NN} n_{i,t+s}^2 + \varpi_{CN} c_{i,t+s} n_{i,t+s}\right] \quad \forall i$$

[15] Sometimes this is referred to as the "risky" steady state (Coeurdacier, Rey, and Winant, 2011) instead.

where $\varpi_{CC} \equiv \frac{1}{2(1-\beta)} \frac{\overline{C}\chi\overline{N}^{\theta}}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2}$, $\varpi_{NN} \equiv \frac{1}{2(1-\beta)} \frac{\overline{C}\chi\theta^2\overline{N}^{\theta}}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2}$, and $\varpi_{CN} \equiv \frac{1}{(1-\beta)} \frac{\overline{C}\chi\theta\overline{N}^{\theta}}{\left(\overline{C}-\chi\overline{N}^{\theta}\right)^2}$. Converting to variances yields

$$= -\varpi_{CC} Var(c_{i,t}) - \varpi_{NN} Var(n_{i,t}) + \varpi_{CN} Cov(c_{i,t}, n_{i,t}) \tag{36}$$

B.2 Calculating the Stochastic Steady State

Kim and Kim (2003) and Woodford (2003) caution that the standard second-order approximation of welfare only yields accurate analysis when alternative policies do not affect the stochastic steady state $\overline{W}$. Policies that affect consumption variance are well known to affect the stochastic steady state in open economy models, but even in closed economy real business cycle models, this channel affects welfare comparisons, as in Cho, Cooley, and Kim (2015) and Heiberger and Maußner (2020). This literature argues that accurately evaluating welfare requires using second order approximations of the equilibrium conditions or other perturbations to measure how variances determine the stochastic steady state. However, these methods quickly become intractable in the present model with dispersed information, endogenous signals, and capital.

Therefore, instead of an analytical approximation, I compute the effects of variances on the stochastic steady state $\overline{W}$ numerically. Approximating models to the second order implies that the stochastic steady state is a function of variance terms:

$$\overline{W} = f(Var(m_{i,t}), Var(r_{N,i,t}), Cov(m_{i,t}, r_{N,i,t})) \tag{37}$$

$\overline{W}$ can be written in this way because it is not a function of the policy parameters directly; they do not affect the deterministic steady state so they only affect the stochastic steady state through their effects on dynamics. The only second order terms that affect the stochastic steady state are variance terms in the expectational equations. In this model, the only expectational equation is the Euler equation (6), which depends on the variances and covariance of the log stochastic discount factor $m_{i,t}$ and the net return on capital $r_{N,i,t}$, which is defined in levels by

$$R_{N,i,t+1} = \frac{R_{K,i,t+1}U_{i,t+1} + Q_{i,t+1}\left(1 - \delta(U_{i,t+1})\right)}{Q_{i,t}} \tag{38}$$

Let $\vec{V}_1$ denote the vector of variance terms affecting the stochastic steady state in equation (37), and let $\vec{V}_2$ denote the vector of variance terms affecting the second order expansion in equation (36). The $*$ superscript denotes values in the full information equilibrium and $\mathbf{1}$ denotes a column vector of ones. The welfare equation (35) can then be written as a quadratic in terms of variances by taking a second order approximation of the stochastic steady state function (37):

$$
\begin{aligned}
&W(\vec{V}_1,\vec{V}_2)\\
&\approx W^* + \frac{\partial SOE}{\partial \vec{V}_1}|'_{\vec{V}_1^*}(\vec{V}_1-\vec{V}_1^*) + \frac{\partial f}{\partial \vec{V}_2}|'_{\vec{V}_2^*}(\vec{V}_2-\vec{V}_2^*) + \frac{1}{2}\mathbf{1}'\frac{\partial^2 f}{\partial \vec{V}_2^2}|_{\vec{V}_2^*}\circ(\vec{V}_2-\vec{V}_2^*)(\vec{V}_2-\vec{V}_2^*)'\mathbf{1}
\end{aligned}
\tag{39}
$$

where $\frac{\partial SOE}{\partial \vec{V}_1}|_{\vec{V}_1^*}$ and $\frac{\partial f}{\partial \vec{V}_2}|_{\vec{V}_2^*}$ are gradients at the full information equilibrium, $\frac{\partial^2 f}{\partial \vec{V}_2^2}|_{\vec{V}_2^*}$ is the Hessian matrix, and $\circ$ denotes the Hadamard product.

The key insight to this numerical method is that the first-best aggregate policy under a full information is to eliminate the information friction entirely. This implies that the full information welfare function of the policy parameters $W^{FI}(b_\nu, b_a, b_z)$ is at its maximum when $b_\nu = b_a = b_z = 0$. Accordingly, the numerical derivative (39) can be calculated by perturbing the vector $\vec{b} = (b_\nu, b_a, b_z)'$ around the origin. For accuracy, the perturbation around the full information point should be symmetric, i.e. perturbing $\vec{b} = \pm\Delta$ rather than in a single direction. Optimality implies:

$$
\begin{aligned}
0 &= \frac{dW(\vec{V}_1,\vec{V}_2)}{d\vec{b}}|_{\vec{b}=0}\\
&= \frac{\partial SOE}{\partial \vec{V}_1}|'_{\vec{V}_1^*}\frac{d\vec{V}_1}{d\vec{b}}|_{\vec{b}=0} + \frac{\partial f}{\partial \vec{V}_2}|'_{\vec{V}_2^*}\frac{d\vec{V}_2}{d\vec{b}}|_{\vec{b}=0} + \frac{1}{2}\sum_{i,j}\frac{\partial^2 f}{\partial \vec{V}_{2,i}\partial \vec{V}_{2,j}}|_{\vec{V}_2^*}\frac{d\vec{V}_{2,i}\vec{V}_{2,j}}{d\vec{b}}|_{\vec{b}=0}
\end{aligned}
$$

where $\vec{V}_{2,i}$ denotes the ith entry in $\vec{V}_2$. The vector $\frac{\partial SOE}{\partial \vec{V}_1}|_{\vec{V}_1^*}$ is known analytically, the square matrices $\frac{\partial \vec{V}_1}{\partial \vec{b}}|_{\vec{b}=0}$ and $\frac{\partial \vec{V}_2}{\partial \vec{b}}|_{\vec{b}=0}$ and the vectors $\frac{d\vec{V}_{2,i}\vec{V}_{2,j}}{d\vec{b}}|_{\vec{b}=0}$ can be found by perturbation, so the vector $\frac{\partial f}{\partial \vec{V}_2}|'_{\vec{V}_2^*}$ and Hessian $\frac{\partial^2 f}{\partial \vec{V}_2^2}|_{\vec{V}_2^*}$ can be calculated.

With the equation (39) derivatives in hand, $W(\vec{V}_1,\vec{V}_2)$ can now be directly compared across policies, by calculating their implied variance vectors $\vec{V}_1$ and $\vec{V}_2$. This computational strategy works by fitting a quadratic approximation of the welfare function around the full information equilibrium. It assumes that full information is optimal (at least relative to the policymaker's possible rules) in order to identify the derivatives of the welfare function, so this welfare approximation ensures that relaxing the information friction is the first-best policy.

B.3 Intertemporal Decomposition

This section describes the decomposition of the welfare function into pre- and post-signal realization components, which are used in Section 4.3 to understand policy trade-offs.

The numerical approach to calculating expected welfare derived in this appendix depends on variances and covariances of several economic variables: the calculation of the stochastic steady state (39) depends on the covariance matrix of stochastic discount and capital return, while the deviation from the stochastic steady state (36) depends on the covariance matrix of consumption and labor. Taken together, express this function of covariances by

$$E\left[W_{i,t}\right] = g(\vec{C})$$

where $\vec{C}$ is a vector of unconditional variances and covariances.

Every covariance can be decomposed into terms that depend on shocks before and after signals are realized into productivity or noise. Consider any two variables x_t^1 and x_t^2 given by lag-operator polynomials $x_t^i = \sum_{j=0}^{\infty} X_j^i L^j \vec{\epsilon}_t$ of the shock vector $\vec{\epsilon}_t$. The covariance is

$$Cov(x_t^1, x_t^2) = \sum_{j=0}^{\infty} X_j^1 Var(\vec{\epsilon}_t)(X_j^2)'$$

The pre-realization component $Cov_{Pre}(x_t^1, x_t^2)$ is the variance due to the first κ periods of responses to shocks, while the post-realization component $Cov_{Post}(x_t^1, x_t^2)$ is the variance due to the remaining component:

$$Cov_{Pre}(x_t^1, x_t^2) = \sum_{j=0}^{\kappa} X_j^1 Var(\vec{\epsilon}_t)(X_j^2)' \qquad Cov_{Post}(x_t^1, x_t^2) = \sum_{j=\kappa+1}^{\infty} X_j^1 Var(\vec{\epsilon}_t)(X_j^2)'$$

Accordingly, the vector of covariances $\vec{C}$ is the sum of vectors of pre- and post-realization covariance components:

$$\vec{C} = \vec{C}_{Pre} + \vec{C}_{Post}$$

Finally, the pre-realization welfare component W_{Pre} is the component that depends only on pre-realization covariances:

$$W_{Pre} \equiv g(\vec{C}_{Pre})$$

while the post-realization is the residual, which includes both post-realization covariances and interaction terms:

$$W_{Post} \equiv g(\vec{C}) - g(\vec{C}_{Pre})$$

so that $E\left[W_{i,t}\right] = W_{Pre} + W_{Post}$.

B.4 Derivation of Consumption Equivalents

This section derives the consumption equivalent welfare costs of business cycles under alternative policies, similar to the strategy used by Lucas (1987) and Lucas (2003).

Consider the expected welfare in the baseline model W_B. Let C_B denote the fixed level of consumption that households would be willing to accept while working the steady state level of hours $\overline{N}$ in order to entirely eliminate macroeconomic volatility. This consumption level solves

$$W_B = \frac{\ln(C_B - \chi \overline{N}^{\theta})}{1-\beta}$$

which is the welfare value implied by equation (34) when consumption and labor are fixed.

The consumption-equivalent cost of volatility in the baseline is the difference between the steady state and this alternative consumption level C_B:

$$\overline{C} - C_B = \overline{C} - e^{(1-\beta)W_B} + \chi \overline{N}^{\theta}$$

Thus the consumption equivalent cost measures how much average consumption a household would be willing to give up in order to eliminate all risk, conditional on continuing to work the same average hours.

Similarly, let C_A denote the corresponding level of consumption under an alternative policy, so that $\overline{C} - C_A$ is the consumption-equivalent cost of volatility in the alternative equilibrium. Measured in consumption equivalents, the welfare gain of moving from the baseline to the alternative equilibrium is $C_A - C_B$, which is given by

$$C_A - C_B = e^{(1-\beta)} \left(e^{W_A} - e^{W_B} \right) \tag{40}$$

C Optimal Noise

This section models the decision of agents who must choose the variance of the noise shocks that they face. I maintain the assumption that agents exactly observe the contemporaneous and past variables that directly affect them, but can pay some cost to reduce the noise in their signals about future productivity. This is modeled as a two-stage decision-making process: first, agents calculate their expected welfare conditional on their noise processes, then agents select the optimal noise process given some cost function.

C.1 Optimal Signal Problem

I modify the process for the signal with idiosyncratic noise (24):

$$\nu_{i,t} = \epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa} + \frac{\bar{s}_i}{\sigma_\zeta} \zeta_t + \frac{\hat{s}_i}{\sigma_{\hat{\zeta}}} \hat{\zeta}_{i,t}$$

Agents on island i control the variables $\bar{s}_i$ and $\hat{s}_i$, which scale the contributions of aggregate and idiosyncratic noise shocks to the signal $\nu_{i,t}$. I modify the signal in this way so that agents can affect the contribution of aggregate noise to their own signal, but cannot directly control the variance of ζ_t which also affects the aggregate economy in equilibrium. The variables $\bar{s}_i$ and $\hat{s}_i$ are scaled by the standard deviations σ_ζ and $\sigma_{\hat{\zeta}}$ so that the variances of aggregate and idiosyncratic noise in the signal are $\bar{s}_i^2$ and $\hat{s}_i^2$ respectively.

The representative household on island i chooses the noise shock variances $\bar{s}_i^2$ and $\hat{s}_i^2$, taking all other agents' choices as given, in order to maximize expected future welfare as given by equation (34), less a utility cost $K\left(\bar{s}_i^2, \hat{s}_i^2\right)$ that is decreasing in the shock variances:

$$\max_{\bar{s}_i^2, \hat{s}_i^2 \geq 0} E\left[W_{i,t}\right] - K\left(\bar{s}_i^2, \hat{s}_i^2\right) \tag{41}$$

As in Maćkowiak and Wiederholt (2015), agents make this choice once for all periods based on their unconditional expectation, then the economy runs forever in the stationary equilibrium associated with the noise choices.

I assume a symmetric cost function $K(\bar{s}_i^2, \hat{s}_i^2)$ so as not to *ex ante* bias agents towards preferring one type of noise over another. The first order conditions for this problem (for an interior solution) are:

$$\frac{\partial E\left[W_{i,t}\right]}{\partial \bar{s}_i^2} = \frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \bar{s}_i^2} \qquad \frac{\partial E\left[W_{i,t}\right]}{\partial \hat{s}_i^2} = \frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \hat{s}_i^2}$$

If these choices only affected welfare by increasing the forecast accuracy of future productivity, a symmetric and convex cost function would imply that agents would choose $\bar{s}_i^2 = \hat{s}_i^2$. Agents would be indifferent about the source of the noise in their signals, and the cost of reducing each type of noise is increasing, so they would reduce noise symmetrically. But the noise shocks are not symmetric: they are equally informative about future productivity, but not about the remaining macroeconomic aggregates that households have incentive to forecast. If other agents choose any weight on aggregate noise at all, then the household will strictly prefer their own noise be aggregate instead of idiosyncratic. This leads households to choose *asymmetric attention* (Kohlhas and Walther, 2021) in equilibrium.

With this asymmetric incentive, an exact form for the cost function $K(\bar{s}_i^2, \hat{s}_i^2)$ must be assumed in order to solve the model. I consider two forms: a standard entropy-based cost which leads to an aggressive weighting on aggregate noise, and a separable cost linear in precisions, which leads to a conservative weighting of the two components.

C.2 Entropy-based Cost

In the standard rational inattention model (Sims, 2003), agents observe a noisy signal, and pay a cost that depends on the mutual information. This structure has a natural mapping to the present setting: agents are observing a noisy signal $\nu_{i,t}$ of their future productivity innovation $\epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa}$. When shocks are Gaussian, the mutual information $I(\epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa}, \nu_{i,t})$ is

$$I(\epsilon_{\hat{a},i,t+\kappa} + \epsilon_{a,t+\kappa}, \nu_{i,t}) = \frac{1}{2}\log_2\left(\frac{\sigma_a^2 + \sigma_{\hat{a}}^2}{\bar{s}_i^2 + \hat{s}_i^2} + 1\right)$$

and agents solving problem (41) pay a cost propotional to the mutual information (by scalar μ):

$$K(\bar{s}_i^2, \hat{s}_i^2) = \mu\frac{1}{2}\log_2\left(\frac{\sigma_a^2 + \sigma_{\hat{a}}^2}{\bar{s}_i^2 + \hat{s}_i^2} + 1\right)$$

This problem has a simple solution because of the collinear way that noise variances enter the mutual information function. Each component of the noise process has the *same marginal cost*:

$$\frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \bar{s}_i^2} = \frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \hat{s}_i^2}$$

This is because aggregate and idiosyncratic noise shocks have the same effect on the informativeness of the signal. So the household's solution is to minimize the variance of whichever noise shock has a more negative effect on marginal welfare. This is always the private shock.

For any total noise variance, the household strictly prefers a process where the contributing shocks are entirely aggregate. This is because both types of noise reduce the households information about productivity, but the aggregate noise shock increases the household's information about aggregate demand for their output. Section 5.1 describes this effect in greater detail, and Figure 8 panel (b) makes the mechanism clear: raising the relative contribution of the aggregate noise shock improves forecasting accuracy for both demand and productivity.

Figure 10 demonstrates that agents strictly prefer lowering the idiosyncratic noise variance $\hat{s}_i^2$ before lowering the aggregate noise variance $\bar{s}_i^2$. The figure plots the difference between the marginal welfares net of their marginal costs:

$$\text{Net Marginal Benefit } = \left(\frac{\partial E\left[W_{i,t}\right]}{\partial \bar{s}_i^2} - \frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \bar{s}_i^2}\right) - \left(\frac{\partial E\left[W_{i,t}\right]}{\partial \hat{s}_i^2} - \frac{\partial K(\bar{s}_i^2, \hat{s}_i^2)}{\partial \hat{s}_i^2}\right)$$

For the entropy-based cost (solid blue line), the marginal costs are equal, so this is just the difference between marginal welfares. The marginal welfare of increasing $\bar{s}_i^2$ is always higher than for $\hat{s}_i^2$, except when the economy-wide weight is $w_\zeta = 0$, at which point agents are indifferent between the sources of their noise. The marginal welfare difference is not continuous at this point (it jumps discretely in Figure 10), and for every other value the curve is strictly positive. This implies that $w_\zeta = 1$ is the equilibrium; given any nonzero amount of noise, agents must select a variance allocation with all weight on the aggregate component.

The entropy-based cost is extreme in it's prediction: the optimal noise problem has a corner solution because the marginal cost of decreasing aggregate vs idiosyncratic noise is always the same. But what if each type of noise has an independently increasing marginal cost? This leads to an interior solution.

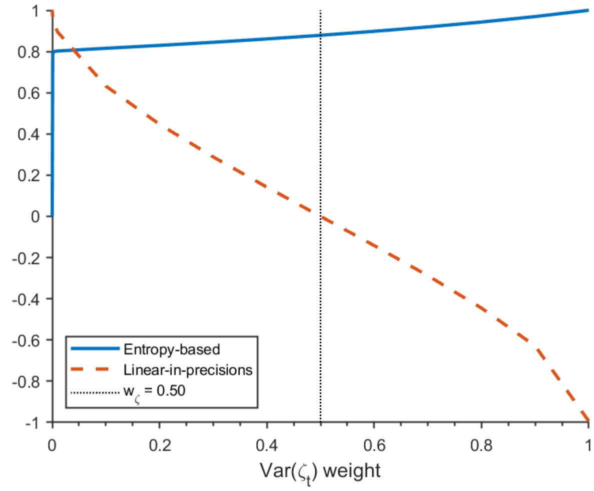

Figure 10: Net Marginal Benefits of $\bar{s}_i^2 - \hat{s}_i^2$ (Normalized)

C.3 Linearity in Precisions

As an alternative, I consider a cost function that is linear in the precision of each noise component:

$$K(\bar{s}_i^2, \hat{s}_i^2) = \phi \frac{1}{\bar{s}_i^2} + \phi \frac{1}{\hat{s}_i^2}$$

with a common coefficient ϕ to maintain the symmetry of each type of noise shock. Linearity in precisions is enough to ensure an interior solution, at least in this model, because the marginal cost is decreasing in shock variances fast enough to maintain single crossing with marginal welfare.

The implied first order conditions are:

$$\frac{\partial E\left[W_{i,t}\right]}{\partial \bar{s}_i^2} = -\phi \frac{1}{\bar{s}_i^2} \qquad \frac{\partial E\left[W_{i,t}\right]}{\partial \hat{s}_i^2} = -\phi \frac{1}{\hat{s}_i^2}$$

Note that the marginal costs are negative, but marginal welfares are as well: increasing signal noise reduces forecastability and welfare.

To find a solution on the w_ζ curve studied in Section 5.1, I calibrate the cost function by implicitly choosing ϕ so that $\bar{s}_i^2 + \hat{s}_i^2 = (0.115)^2$. The values that both satisfy this equation and the combined first order conditions $\frac{\partial E[W_{i,t}]}{\partial \bar{s}_i^2} \bar{s}_i^2 = \frac{\partial E[W_{i,t}]}{\partial \hat{s}_i^2} \hat{s}_i^2$ correspond to an aggregate noise weight of $w_\zeta = 0.50$, just above one half. This can be seen in Figure 10, where the net marginal benefit corresponding to the linear cost function (dashed red line) intersects zero at 0.50. This implies that $w_\zeta = 0.50$ is the equilibrium; this is the interior solution where an agent's optimal individual allocation of aggregate vs. idiosyncratic noise matches the economy-wide allocation.

This quantitative solution is specific to the linear cost function chosen. But more generally, if the cost function is symmetric and convex in precisions, agents will always choose a noise allocation corresponding to $w_\zeta \geq 0.50$, because they strictly prefer to reduce the idiosyncratic component of their noise relative to the aggregate component.

D Noise Shock Estimation

This section describes how I estimate the idiosyncratic noise process in Section 5.1 by adapting the Chahrour and Jurado (2022) procedure to industry-level data.

D.1 Identification

Let $a_{i,t}$ denote log TFP in industry i at time t. a_t denotes aggregate log TFP, and $\hat{a}_{i,t}$ denotes the orthogonal component of industry log TFP, so that $a_{i,t} = a_t + \hat{a}_{i,t}$. I use lower-case variables to differentiate from the explicitly modeled processes in Section 2. Instead, the components of TFP are assumed to have general autoregressive processes:

$$a_t = \sum_{j=1}^{n_a} B_{a,j} a_{t-j} + \epsilon_{a,t} \qquad \hat{a}_{i,t} = \sum_{j=1}^{n_{\hat{a}}} B_{\hat{a},j} \hat{a}_{i,t-j} + \epsilon_{\hat{a},i,t} \tag{42}$$

where $\epsilon_{a,t}$ and $\epsilon_{\hat{a},i,t}$ are i.i.d. white noise shocks.

Let b_t and $\hat{b}_{i,t}$ denote κ-period-ahead forecasts, conditional on vectors of time series y_t and $\hat{y}_{i,t}$ and TFP:

$$b_t = E[a_{t+\kappa} | \{y_{t-j}, a_{t-j}\}_{j=0}^{n_a}] \qquad \hat{b}_{i,t} = E[\hat{a}_{i,t+\kappa} | \{\hat{y}_{i,t-j}, \hat{a}_{i,t-j}\}_{j=0}^{n_{\hat{a}}}] \tag{43}$$

The time series $\hat{y}_{i,t}$ is orthogonalized with respect to aggregate time series y_t, so $\hat{b}_{i,t}$ and b_t are necessarily orthogonal as well.

The main identifying assumption is that absent any news about future TFP, equation (42) implies that the forecast would only depend on current and past productivities, with no weight on the other time series. Thus the orthogonalized forecasts $b_t^\perp$ and $\hat{b}_{i,t}^\perp$ capture the contribution of news and noise to expectations:

$$b_t^\perp = E[a_{t+\kappa} | \{y_{t-j}, a_{t-j}\}_{j=0}^{n_a}] - E[a_{t+\kappa} | \{a_{t-j}\}_{j=0}^{n_a}]$$
$$\hat{b}_{i,t}^\perp = E[\hat{a}_{i,t+\kappa} | \{\hat{y}_{i,t-j}, \hat{a}_{i,t-j}\}_{j=0}^{n_{\hat{a}}}] - E[\hat{a}_{i,t+\kappa} | \{\hat{a}_{i,t-j}\}_{j=0}^{n_{\hat{a}}}] \tag{44}$$

The components of $b_t^\perp$ and $\hat{b}_{i,t}^\perp$ that are also orthogonal to future productivity capture noise alone. For horizons m_a and $m_{\hat{a}}$, the noise components b_t^* and $\hat{b}_{i,t}^*$ are given by

$$b_t^* = b_t^\perp - E[b_t^\perp | \{a_{t+j}\}_{j=1}^{m_a}] \qquad \hat{b}_{i,t}^* = \hat{b}_{i,t}^\perp - E[\hat{b}_{i,t}^\perp | \{\hat{a}_{i,t+j}\}_{j=1}^{m_{\hat{a}}}] \tag{45}$$

Finally, the noise components may be autocorrelated, so they are written in terms of the i.i.d. noise shocks ζ_t and $\hat{\zeta}_{i,t}$:

$$b_t^* = \sum_{j=1}^{n_b} B_{b,j} b_{t-j}^* + \zeta_t \qquad \hat{b}_{i,t}^* = \sum_{j=1}^{n_{\hat{b}}} B_{\hat{b},j} \hat{b}_{i,t-j}^* + \hat{\zeta}_{i,t} \tag{46}$$

Noise shocks are identified by estimating equations (43), (44), (45), and (46) by OLS.

D.2 Data

Chahrour and Jurado (2022) estimate their VAR using 7 time series at quarterly frequencies. When estimating the model, I attempt to follow their choices as closely as possible, but some changes are unavoidable. First, I have to use annual instead of quarterly data, because quarterly TFP measures are unavailable or poor at the industry level. Second, while some of their time series have clear available analogs at the industry level, others do not, so the industry forecasts employ fewer time series.

Industry data are from the NBER-CES Manufacturing Industry Database (Becker, Gray, and Marvakov, 2021) aggregated at the 4-digit SIC code level. This dataset contains estimates from 1958-2018 of inputs, output, and TFP estimated using capital, production labor, non-production labor, energy, and materials. Unlike in the aggregate data, TFP is not directly adjusted for capital utilization, although utilization is correlated with other inputs besides labor. The other source for industry data is CRSP Stocks from which I construct indices of equity returns at the 4-digit SIC industry level. Table 4 lists the remaining time series used for estimation. Following Chahrour and Jurado (2022), all time series are differenced once before estimation to remove unit roots in TFP and trends more generally.

	Series Name	Source
Aggregate Series	Log Utilization-Adjusted TFP	SF Fed (Fernald, 2014)
	Log Real GDP	NIPA
	Log Real Non-Durable Consumption	NIPA
	Inflation (GDP Deflator)	NIPA
	Log Hours	NIPA
	3-Month Nominal T-Bill Yields	Federal Reserve
	Log Real Value-weighted Stock Price Index	CRSP
Industry-specific Series	Log TFP	NBER-CES
	Log Real Output	NBER-CES
	Log Hours (All Employees)	NBER-CES
	Log Real Value-weighted Stock Price Index	CRSP

Table 4: Time Series Used for Noise Shock Estimation

D.3 Estimates

When estimating, I set $n_b = n_{\hat{b}} = 2$; Chahrour and Jurado (2022) use four quarters, but a single lagged year is not enough to whiten the noise shocks. Otherwise, I map their remaining parameters to an annual frequency as closely as possible: $\kappa = 5$, $m_a = m_{\hat{a}} = 5$, and $n_a = n_{\hat{a}} = 3$.

Table 5 reports the estimated noise shock variances from several specifications. For aggregate noise, I run the regressions using all seven time series, as well as only the subset of four that have industry-specific analogs (TFP, output, hours, and the stock price index). I also run these specifications using both the full sample and the short sample that matches the same years for which industry-specific estimates are available. The results are broadly similar across these specifications. I choose the long sample with the four common series as the preferred specification (in bold).

For the industry-specific noise, I run three specifications that implied more varied results. My preferred estimate (in bold) pools all industries, which is most resembles the economic model. A specification including industry-specific fixed effects implies lower industry-specific noise variance, but is exposed to the standard concerns when estimating dynamic panel regressions with fixed effects.[16] I also perform the entire estimation separately by industry, aggregating at the 2-digit SIC level, and using only industries with at least 30 observations. This specification implies the highest idiosyncratic noise variance. This dynamic panel regression features additional concerns specific to this estimation procedure, because any measurement error that affects the econometrician but not firms will overstate the effects of noise shocks.

	Noise Shock Variance	Baseline-Implied w_ζ	Shock Observations	Sample Period
Aggregate Estimates				
Common 4 series	$\mathbf{(0.0049)^2}$		**65**	**1955-2019**
Common 4 series (short sample)	$(0.0051)^2$		48	1966-2013
Full 7 series	$(0.0050)^2$		65	1955-2019
Full 7 series (short sample)	$(0.0050)^2$		48	1966-2013
Industry-specific Estimates				
Pooled	$\mathbf{(0.0051)^2}$	**0.48**	**10,561**	**1966-2013**
Industry fixed effects	$(0.0034)^2$	0.67	10,561	1966-2013
Pooled within 2-digit SIC	$(0.0084)^2$	0.30	10,561	1966-2013

Table 5: Shock Variance Estimates

In order to calibrate the aggregate noise weight w_ζ that achieve the baseline calibrations for total noise variance (Table 1), I look for a common scaling factor λ_ζ that solves

$$Var(\zeta_{i,t}) = \lambda_\zeta Var(\zeta_t) + \lambda_\zeta Var(\hat{\zeta}_{i,t})$$

for the estimates $Var(\zeta_t)$ and $Var(\hat{\zeta}_{i,t})$, and the calibrated value $Var(\zeta_{i,t}) = (0.115)^2$. To map back to the noise-weighting representation in equation (25), the aggregate

[16] Nickell bias (Nickell, 1981) in dynamic panel regressions is known to attenuate coefficient estimates, but it is not clear if the effect on the estimated noise variance is attenuating or amplifying.

noise weight w_ζ is given by

$$w_\zeta = \frac{\lambda_\zeta Var(\zeta_t)}{\lambda_\zeta Var(\zeta_t) + \lambda_\zeta Var(\hat{\zeta}_{i,t})}$$

Table 5 reports these weights in the third column, by industry-specific specification. In each case, the weight uses estimated aggregate variance from the preferred specification – the long sample with the four common series.

E Idiosyncratic Noise and Policy Incentives

What are the incentives to enact a countervailing policy rule?

Figure 11 plots the marginal welfare (solid blue line) of increasing the policy elasticity b_ν against the aggregate noise weight of equation (25). In all cases, the marginal elasticity is negative (normalized relative to the $w_\zeta = 1$ baseline): decreasing the elasticity b_ν of the investment wedge to the aggregate signal is welfare improving. This matches the conclusion from the baseline calibration that policy should "lean against the wind." Qualitatively, this conclusion is independent of w_ζ; whether the aggregate component of noise is large or small, agents will over-invest in response to it, so the policymaker can improve welfare by discouraging investment.

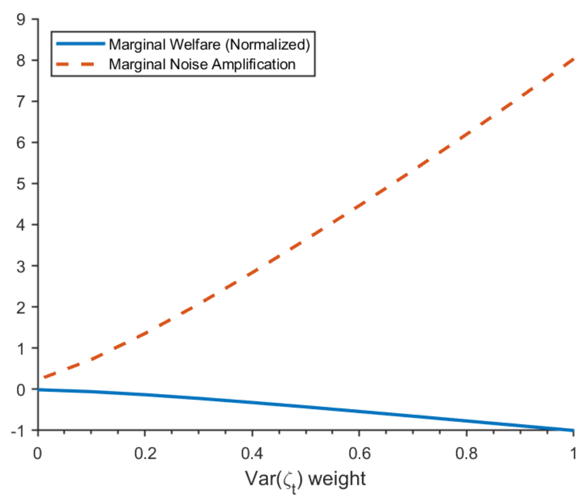

Figure 11: Marginal Policy Rule Effects with Idiosyncratic Noise

The aggregate noise weight w_ζ does matter quantitatively. The marginal welfare with respect to the policy parameter decreases in magnitude as the aggregate noise weight shrinks. This occurs because aggregate noise shocks are rarer; changing the elasticity b_ν has a smaller effect on the variances of other macroeconomic aggregates. Figure 11 demonstrates this channel by plotting the marginal effect of the policy elasticity b_ν on the output amplification in the model (dashed red line). When w_ζ is large, the aggregate noise variance is large and a given b_ν will have a large effect on output volatility. Conversely, when w_ζ is small, the same elasticity b_ν will have a small effect on output volatility, shrinking to zero as the aggregate noise component disappears.